# Decision Models for Corporate Sustainability

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Business Administration
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ABSTRACT

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# Abstract

This dissertation explores decision problems faced by organizations willing to address or support the incorporation of sustainability aspects on their “business as usual” activities. We study to specific problems.

First, we analyze the decision problem of a forest manager who, in addition to selling timber, has the option of selling carbon offsets for the carbon sequestered by the forest. We study both the single-rotation and the multiple-rotations harvesting problems, and develop stochastic dynamic programming models to find the optimal harvesting and offset-selling policy, the expected optimal harvesting time, and the expected optimal reward under different offset-trading schemes.

Then, we study the case in which an organization (sustainability buyer) outsources sustainability efforts to another organization (sustainability seller). While buyers cannot directly exert sustainability efforts, they can provide economic or technical support to their sellers in order to incentivize these efforts. We investigate how the effort and support decisions change according to characteristics of stakeholders, buyers, and sellers. Considering that buyers can compete on the sustainability effort exerted by their sellers, we extend our analysis to the case of competing buyers, and we determine conditions under which sharing sellers is preferred by the buyers to having separate sellers for each buyer.

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# 1. Introduction

Sustainability is an overused work. According to the World Business Council for Sustainable Development there are over 100 definitions of sustainability and sustainable development[1]. One of the most popular is that of United Nations, which defines sustainable development as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs."[2] For corporations, this implies doing business considering their economic, social, and environmental impacts.

This idea of corporate sustainability has received significant attention in the last years from both companies and stakeholders. Countries have adopted international agreements such as the Kyoto Protocol to improve their overall environmental performance. Investors want to select projects and companies that consider not only the economic but also the social and environmental dimensions. Consumers are demanding products manufactured and services offered in a sustainable way. As a response to stakeholder pressure, companies are changing their approach to sustainability issues, from isolated social and environmental projects to corporate sustainability strategies and practices that are part of their core business, create value, and give them a competitive advantage.

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[1] http://www.wbcsd.ch/web/course/gsc/web/glossary/s.htm (accessed March 24, 2013)
[2] http://www.un-documents.net/ocf-02.htm

The social and environmental impact of business operations have been studied for a long time. However, the main focus has been on surveys and case studies about companies' goals, perception, and practices regarding sustainability. Considering the importance that sustainability issues can have on the core business and strategies of the companies, our goal is to develop decision frameworks that companies and stakeholders can use to make better decisions that foster the implementation of sustainability initiatives and improve companies' sustainability performance. Sustainability decision problems usually encompass multiple objectives, multiple stakeholders, and uncertainty about the preferences and the importance given by different parties to these sustainability objectives. Therefore, our decision frameworks rely on models and methodologies from disciplines such as optimization, stochastic processes, and game theory.

We study two specific decision problems faced by different type of organizations willing to address or support the incorporation of sustainability aspects on their "business as usual" activities. We start in Chapter 2, by studying the forest harvesting problem when carbon offsets-trading is considered. Forests' capacity for carbon sequestration may bring new benefits (or costs) that cause a change in the optimal harvesting policies. We analyze the decision problem of a forest manager who, in addition to selling timber, has the option of selling carbon offsets for the carbon sequestered by the forest. We study both the single-rotation and the multiple-rotations harvesting problems, and develop stochastic dynamic programming models to find the optimal harvesting and offset-selling policy, the expected optimal harvesting time, and the expected optimal reward under different timber and carbon price scenarios. We compare these results with those obtained under a subsidies-tax scheme, where the forest manager receives a subsidy for the carbon sequestered in each period and then pays a harvesting tax. We then provide recommendations for forest managers and social planners regarding the offset-trading scheme that can best help them to meet their goals.

In Chapter 3, we move to a more general problem faced by different type of organizations (e.g. companies, governments, communities) that are interested in leading sustainability initiatives within the system where they operates but may not have the capacity or control required to implement these initiatives directly. Specifically, we consider the situation in which a buyer outsources sustainability efforts to a seller. Stakeholders put pressure on the buyer to improve their sustainability performance. While buyers cannot directly exert sustainability efforts, they can provide economic or technical support to their sellers in order to incentivize sustainability efforts. We investigate how the effort and support decisions change according to characteristics of stakeholders, buyers, and sellers. Our framework allows us to incorporate not only cost reduction but also many of the other benefits that may drive sustainability efforts such as corporate image improvement, regulatory compliance, and community relations improvement.

Buyer's and seller's decisions may have an impact on competing buyers and sellers. To capture this impact, we extend our basic buyer-seller model to consider the case of two buyers who compete on sustainability performance in two possible network structures. In the first structure, the two buyers share the same seller, while in the second structure the two buyers have separate sellers. We determine conditions on stakeholder, buyer, and seller characteristics that may lead buyers and sellers to prefer one network structure over the other.

# 2. Impact of Carbon Offsets Trading on Optimal Forest Harvesting

## *2.1 Motivation and Literature Review*

Timber harvesting is probably the most studied topic in forest economics. According to Samuelson (1976), the German forester Martin Faustman was the first to propose an "appropriate model" for determining the optimal time to harvest a forest stand (an area of a forest with similar properties). Faustman's model finds the rotation age that maximizes the net present value (NPV) of the benefits from an infinite number of identical rotations, starting from bare land. Although Faustman's model is built on a set of restrictive assumptions such as a homogeneous forest, a constant forest management program, and deterministic variables, it has served as the basis for many other harvesting decision models formulated to find the optimal harvesting policy (OHP) in more realistic settings.

Extensions to Faustman's model studied different aspects of the harvesting decision problem. For example, some of these models incorporated management activities such as thinning and regeneration in the set of decision variables (see e.g. Brodie and Haight 1985, Brazee and Bulte 2000, Gong and Löfgren 2009). Other models have studied the impact of non-timber benefits such as carbon sequestration, water supply, and biodiversity value on the OHP (see e.g. Hartman 1976, Alaouze 2004, Spring et. al. 2005, Gutrich and Howarth 2007). Nevertheless, the most widely-studied aspect has been the impact of economic, biological, and ecological uncertainty on the OHP.

Brazee and Mendelsohn (1988), Haight and Holmes (1991), and Plantinga (1998) are good examples of papers that analyze the impact of timber price uncertainty. Van Kooten et. al. (1992) develop a model where the trees' growth is uncertain and is also affected by forest management decisions. Furthermore, Clarke and Reed (1989) incorporate both price uncertainty and forest growth uncertainty into their harvesting model. Finally, Spring et. al. (2005) propose a model to study the impact of climate change effects, particularly forest fire increase, on the OHP.

Despite advances in the study of the OHP, some aspects can still be improved. Gong and Löfgren (2009) point out the integrated analysis of uncertainty and non-timber benefits as one of the topics that deserves more attention. They note that most of the harvest decision models that incorporate uncertainty ignore the non-timber benefits of the forest, whereas models including multiple forest benefits usually assume certainty on economic, biological and ecological variables. In particular, one of the non-timber benefits that has received more attention is a forest's carbon dioxide (CO2) sequestration capability.

The demand for carbon offsets (defined as the reduction in emissions of one metric ton of CO2 or its equivalent in other greenhouse gases (GHGs) as specified by the Kyoto protocol ) represents an additional business opportunity for forest managers. Forest carbon reduction projects are usually classified as reforestation, avoided conversion of forest land to non-forest land, and modified forest management to increase the amount of carbon sequestered by the forest. However, the specific mechanisms to define the payments associated to these offsets are still in their early stages, and a unified protocol does not exist. Forest offsets have generally been sidelined in the compliance markets, and mainly commercialized in the voluntary market. In this market, projects are usually evaluated and certified by third parties, such as the Climate Action Reserve (CAR) and the Voluntary Carbon Standard (VCS), to increase their value. Nevertheless, the size of the forest offsets market has significantly increased over the last years. According to a 2009 study developed by Ecosystem Marketplace across 40 countries, the volume of forest offsets transacted went from 0.8 million tonnes of carbon dioxide (MtCO2) in 2005, with an estimated value of $4.3 million, up to 5.3 MtCO2 in 2008, with an estimated value of $37.1 million.

The increasing interest on the role of the forests as carbon sinks has motivated researchers to study how the OHP changes when forest's capacity for carbon sequestration is incorporated into the analysis. In an early attempt, van Kooten et. al. (1995), proposed a deterministic model to explore the impact of carbon sequestration on the harvesting decision. In their model, the carbon sequestered at any point on time is proportional to the merchantable timber volume. The forest manager receives payments for the carbon sequestered in each period and pays a tax (i.e. a reversal penalty) for the carbon released at the time of harvest. Gutrich and Howarth (2007) extended this model by considering the carbon stored not only in live biomass but also in dead biomass, soil and wood products. Finally, Chladná (2007) uses a carbon sequestration and commercialization setting similar to the one defined by van Kooten et. al. (1995), but incorporates uncertainty in the timber and carbon prices. She developed a real-option model to analyze the expected optimal rotation length under different values of the reversal penalty.

The works previously mentioned incorporated carbon sequestration through a subsidies-tax scheme where the forest manager receives a subsidy for the carbon sequestered in each period and then pays a harvesting tax. In this chapter, we consider a more market-based mechanism where the forest manager, in addition to selling timber, has the option of selling carbon offsets at any given period. We study both the single-rotation and the multiple-rotations harvesting problems and develop stochastic dynamic programming models to find the optimal harvesting and offset-selling policy, the expected optimal harvesting time, and the expected optimal reward under different price scenarios. We compare these results with those obtained under a subsidies-tax scheme. We also study the impact of the so-called "additionality constraint" imposed by most offset-certifying organizations (see e.g. The Climate Action Reserve's protocol, 2010), which states that offsets can be sold at a given period only if they represent CO2 that would not have been sequestered under "business as usual" (e.g. if business as usual would have harvested the stand).

The chapter is organized as follows. In section 1, we describe the four different offset-trading schemes studied in the chapter: a base case without offsets, a subsidies-tax scheme, optional offset selling, and optional offset selling with the additionality constraint. In section 2, we present the dynamic programming models formulated to find the optimal harvesting and offset-selling policies under these schemes. We develop models for both the finite-horizon single-rotation problem and the infinite-horizon multiple-rotations problem. In section 3, we describe the stochastic processes of timber and carbon prices, as well as the approximations of these processes used to numerically solve the dynamic programming models. We then present the results for the finite-horizon and infinite-horizon problems in sections 4 and 5, respectively. We discuss some properties of the optimal harvesting and offset-selling policy, and compare the expected optimal harvesting time and the expected optimal reward under the four schemes for different price scenarios. Finally, we summarize the results and present some insights for forest managers and social planners.

## 2.2 Harvesting Models

We consider a risk-neutral forest manager who must decide the optimal time to harvest a forest stand. In each period, she observes the timber and carbon prices along with timber volume and the carbon stored on the stand. As in van Kooten et. al. 1995 and Chladná 2007, we assume that the carbon stored by the trees at a given time is proportional to the timber volume. She then decides whether to harvest the stand or postpone the harvest and continue growing the stand.

We study both the single-rotation and the multiple-rotations problems. In the single-rotation problem, we initially assume that the forest manager must harvest and sell the land at period $T$ or earlier, and that the value of the bare land is zero (We can think of this assumption as the forest land being under a lease contract that expires after $T$ periods without a renewal option.). We then relax the mandatory "harvest and sell" assumption, giving the forest manager the option of never harvest the stand. In the multiple-rotations problem, we allow the forest manager to replant the forest after every harvest, starting a new rotation.

In addition to the harvesting decision, the forest manager may also decide whether or not to sell carbon offsets, depending on the specific offset-selling scheme under which the forest operates. We consider four different schemes: Scheme I, which does not allow any carbon offset trading; Scheme II, a subsidies-tax scheme similar to the studied by van Kooten et. al. (1995) and Chladná (2007); Scheme III, which gives the forest manager the option of selling carbon offsets in each period and then paying a reversal penalty at the time of harvesting; and Scheme IV, which is similar to Scheme III but adds the additionality constraint.

### 2.2.1 Scheme I: No Offsets

Scheme I does not allow the manager to sell offsets, so it can be considered the business-as-usual scenario. In each period, the forest manager observes the timber volume, $x_t \in \mathbb{R}^{1+}$, and the timber price, $P_t \in \mathbb{R}^{1+}$ (or equivalently the log of the timber price, $y_t \in \mathbb{R}^1$), and makes a harvesting decision, $h_t^I$. We assume that partial harvesting is not feasible, thus $h_t^I \in \{0,1\}$ with $h_t^I = 1$ if harvesting is performed and $h_t^I = 0$ otherwise. Note that if the profit function is linear in $x_t$ as is the case in this analysis, the optimal $h_t^I$ will be either $0$ or $1$.

The timber volume evolves according to the discrete-time system equation

$$x_{t+1}(x_t, h_t^I) = \begin{cases} x_t + f\left(x_t\right) & if\ h_t^I = 0, \\ x_0 & if\ h_t^I = 1, \end{cases} \qquad (2.1)$$

where $f\left(x_t\right)$ represents a increasing concave timber growth function, with $f\left(0\right) = 0$. Note that for the finite-horizon model $x_0 = 0$. The timber price, $P_t$, evolves according to a given stochastic process, which we can write in terms of the log of the price as $\tilde{y}_{t+1}\left(y_t\right) = \tilde{g}\left(y_t\right) = by_t + \tilde{z}_t$, where $b$ is a constant and $\tilde{z}_1, \ldots, \tilde{z}_t, \ldots$ are independent random variables. Note that both Brownian motion and mean-reverting processes satisfy this independent additive-increments condition. More details about the timber price behavior will be provided in the next section.

Under these assumptions, the forest manager's optimization problem consists of determining the harvesting policy, $\{h_t^I(x_t, P_t)\}_{t=1}^T$, that maximizes her reward function

$$E_{\{P_t\}_{t=1}^T}\left[\sum_{t=1}^{T}\left(\frac{1}{1+r}\right)^{t-1} g_t^I\left(x_t, P_t, h_t^I\right)\right] \qquad (2.2)$$

where

$$g_t^I\left(x_t, P_t, h_t^I\right) = \begin{cases} 0 & \text{if } h_t^I = 0, \\ \left(1-c\right)P_t x_t & \text{if } h_t^I = 1, \end{cases} \qquad (2.3)$$

$c$ is the unit harvesting cost, defined as a percentage of the price, and $r$ is the forest manager's discount rate.

For the finite-horizon problem, harvesting is mandatory in the last period. Therefore, a dynamic program (DP) to solve this optimization problem is

$$V_T^I\left(x_T, P_T\right) = \left(1-c\right)P_T x_T,$$

$$V_t^I\left(x_t, P_t\right) = \max\left\{g_t^I\left(x_t, P_t, 1\right), \left(\frac{1}{1+r}\right) E_{P_{t+1}^L}\left[V_{t+1}^I\left(x_{t+1}, P_{t+1}\right) \middle| P_t\right]\right\}. \qquad (2.4)$$

For the multiple-rotations infinite-horizon problem, the forest manager will replant the forest after every harvest, paying a replanting cost $R$. We model the multiple-rotations problem as an infinite-horizon discounted problem, with a discount factor $\delta = e^{-r\Delta t}$. Therefore the corresponding Bellman equation is given by

$$J^*\left(x, y\right) = \max\left\{\delta E\left[J^*\left(f\left(x\right), \tilde{g}\left(y\right)\right)\right], (1-c)e^y x - R + \delta E\left[J^*\left(x_0, \tilde{g}\left(y\right)\right)\right]\right\}. \qquad (2.5)$$

For analytical convenience, we use the log of the timber price, $y$, in our infinite-horizon formulation.

### 2.2.2 Scheme II: Subsidies-tax

In Scheme II the forest manager receives payments for the carbon sequestered by the forest in each period. The payment received at period $t$ will depend on the carbon price, $P_t^C$ (or equivalently the log of the carbon price, $y_t^C \in \mathbb{R}^1$) as well as the carbon sequestered by the forest at that period, which is a proportion, $\gamma$, of the increment in the timber volume, $x_t - x_{t-1}$. At the time of harvesting, the forest manager must pay a tax for the carbon released back to the atmosphere. This payment will be determined by the carbon stored in the forest at period $t$, $\gamma x_t$, the carbon price at that period, and a penalty factor, $\kappa \in [0,1]$. A penalty factor equal to one is similar to assuming that 100% of the carbon sequestered in the trees is released back to the atmosphere at the time of harvesting. However, if we consider the fact that some carbon remains sequestered in the wood products and forest residues after harvesting, then a penalty factor smaller than one may be more appropriate.

We assume that the carbon price evolves according to the stochastic process $\tilde{y}_{t+1}^C\left(y_t^C\right) = \tilde{h}\left(y_t^C\right) = dy_t^C + \tilde{w}_t$, where $d$ is a constant and $\tilde{w}_1, \ldots, \tilde{w}_t, \ldots$ are independent random variables. If the carbon and timber prices are not independent, we can use the conditional probability distribution of $\tilde{w}_t$ given $\tilde{y}_t$ to determine the carbon price transitions.

Given the carbon payments structure, the forest manager's optimization problem under Scheme II consists of determining the harvesting policy, $\{h_t^{II}(x_t, P_t, P_t^C)\}_{t=1}^T$, that maximizes her reward function

$$E_{\{P_t\}_{t=1}^T, \{P_t^C\}_{t=1}^T} \left[ \sum_{t=1}^{T} \left( \frac{1}{1+r} \right)^{t-1} g_t^{II}\left( x_t, P_t, P_T^C, h_t^{II} \right) \right] \qquad (2.6)$$

where

$$g_t^{II}\left( x_t, P_t, P_t^C, h_t^{II} \right) = \begin{cases} P_t^C \gamma \left( x_t - x_{t-1} \right) & \text{if } h_t^{II} = 0, \\ \left( 1-c \right) P_t x_t + P_t^C \gamma \left( x_t - x_{t-1} \right) - P_t^C \kappa \gamma x_t & \text{if } h_t^{II} = 1. \end{cases}$$

The corresponding DP algorithm for the finite-horizon problem is

$$V_T^{II}\left( x_T, P_T, P_T^C \right) = \left( 1-c \right) P_T x_T + P_T^C \gamma \left( x_T - x_{T-1} \right) - P_T^C \kappa \gamma x_T,$$

$$V_t^{II}\left( x_t, P_t, P_t^C \right) = \max \left\{ \begin{array}{l} g_t^{II}\left( x_t, P_t, P_t^C, 1 \right), g_t^{II}\left( x_t, P_t, P_t^C, 0 \right) \\ + \left( \dfrac{1}{1+r} \right) E_{P_{t+1}^L, P_{t+1}^C} \left[ V_{t+1}^{II}\left( x_{t+1}, P_{t+1}, P_{t+1}^C \right) \middle| P_t, P_t^C \right] \end{array} \right\}. \qquad (2.7)$$

For the infinite-horizon model the DP equation becomes

$$J^*\left( x, y, y^c \right) = \max \left\{ \begin{array}{l} e^{y^c} \gamma \left( x - f^{-1}\left( x \right) \right) + \delta E\left[ J^*\left( f\left( x \right), \tilde{g}\left( y \right), \tilde{h}\left( y^c \right) \right) \right], \\ (1-c) e^{y} x - R + e^{y^c} \gamma \left( x - f^{-1}\left( x \right) \right) - \kappa e^{y^c} \gamma x + \\ \delta E\left[ J^*\left( x_0, \tilde{g}\left( y \right), \tilde{h}\left( y^c \right) \right) \right] \end{array} \right\} \qquad (2.8)$$

### 2.2.3 Scheme III: Optional Offset Selling

Scheme III gives the forest manager the option of selling the carbon offsets available in each period. The number of offsets available at period $t$ is calculated as the difference between the carbon stored in the forest at period $t$, $\gamma x_t$, and the number of offsets sold in the previous periods, $z_t$. We assume that the offsets must be permanent (i.e. the length of the offsets contract is infinite), thus at the time of harvesting the forest manager must pay back for the offsets previously sold, according to the current carbon price and the reversal penalty factor, $\kappa$.

Note that if the reward associated with selling offsets is linear in the number of offsets sold (as is the case in this analysis), then selling a proportion of the offsets available is never optimal. Therefore, the forest manager will either sell all the offsets available or none of them, i.e., $u_t^{III} \in \{0, \gamma x_t - z_t\}$. Furthermore, the cumulative carbon offsets sold follows the discrete-time system equation

$$z_{t+1} = \begin{cases} z_t & \text{if } h_t^{III} = 0,\ u_t^{III} = 0, \\ \gamma x_t & \text{if } h_t^{III} = 0,\ u_t^{III} = \gamma x_t - z_t, \\ \gamma x_0 & \text{if } h_t^{III} = 1. \end{cases} \qquad (2.9)$$

The forest manager's optimization problem under Scheme III consists of determining the harvesting and offset-selling policies, $\{h_t^{III}(x_t, P_t, P_t^C, z_t)\}_{t=1}^{T}$ and $\{u_t^{III}(x_t, P_t, P_t^C, z_t)\}_{t=1}^{T}$ that maximizes her reward function

$$E_{\{P_t\}_{t=1}^T,\{P_t^C\}_{t=1}^T}\left[\sum_{t=1}^{T}\left(\frac{1}{1+r}\right)^{t-1} g_t^{III}\left(x_t,P_t,P_T^C,z_t,h_t^{III},u_t^{III}\right)\right], \tag{2.10}$$

where

$$g_T^{III}\left(x_T,P_T,P_T^C,z_T,u_T^{III}\right)=\left(1-c\right)P_T x_T+P_T^C u_T^{III}-P_T^C\kappa(z_T+u_T^{III})$$

$$g_t^{III}\left(x_t,P_t,P_t^C,z_t,h_t^{III},u_t^{III}\right)=\begin{cases}P_t^C u_t^{III}, & \text{if } h_t^{III}=0\\ \left(1-c\right)P_t x_t+P_t^C u_t^{III}-P_t^C\kappa\left(z_t+u_t^{III}\right), & \text{if } h_t^{III}=1\end{cases}$$

The corresponding DP algorithm for the finite-horizon problem is

$$V_T^{III}\left(x_T,P_T,P_T^C,z_T\right)=\max\begin{Bmatrix}\left(1-c\right)P_T x_T-P_T^C\kappa z_T,\\ \left(1-c\right)P_T x_T+P_T^C u_T^{III}-P_T^C\kappa\gamma x_T\end{Bmatrix},$$

$$V_t^{III}\left(x_t,P_t,P_t^C,z_t\right)=\max\begin{Bmatrix}g_t^{III}\left(x_t,P_t,P_t^C,z_t,1,0\right),\\ g_t^{III}\left(x_t,P_t,P_t^C,z_t,1,\gamma x_T-z_T\right),\\ g_t^{III}\left(x_t,P_t,P_t^C,z_t,0,0\right)\\ +\left(\frac{1}{1+r}\right)E_{P_{t+1},P_{t+1}^C}\left[V_{t+1}^{III}\left(x_{t+1},P_{t+1},P_{t+1}^C,z_t\right)\middle|P_t,P_t^C\right],\\ g_t^{III}\left(x_t,P_t,P_t^C,z_t,0,\gamma x_T-z_T\right)\\ +\left(\frac{1}{1+r}\right)E_{P_{t+1},P_{t+1}^C}\left[V_{t+1}^{III}\left(x_{t+1},P_{t+1},P_{t+1}^C,\gamma x_t\right)\middle|P_t,P_t^C\right]\end{Bmatrix}. \tag{2.11}$$

Note that the model for Scheme II is a particular case of the model for Scheme III where $u_t^{III}=\gamma x_t-z_t$ for each period $t$. For the infinite-horizon case, the DP equation is given by

$$J^*\left(x,z,y,y^c\right)=\max\left\{\begin{array}{l}\delta E\left[J^*\left(f\left(x\right),z,\tilde{g}\left(y\right),\tilde{h}\left(y^c\right)\right)\right],\\ e^{y^c}\left(\gamma x-z\right)+\delta E\left[J^*\left(f\left(x\right),\gamma x,\tilde{g}\left(y\right),\tilde{h}\left(y^c\right)\right)\right],\\ (1-c)e^y x-R-\kappa e^{y^c}z+\delta E\left[J^*\left(x_0,\gamma x_0,\tilde{g}\left(y\right),\tilde{h}\left(y^c\right)\right)\right],\\ (1-c)e^y x-R+e^{y^c}\left(\gamma x-z\right)-\kappa e^{y^c}\gamma x+\\ \delta E\left[J^*\left(x_0,\gamma x_0,\tilde{g}\left(y\right),\tilde{h}\left(y^c\right)\right)\right]\end{array}\right\} \qquad (2.12)$$

### 2.2.4 Scheme IV: Optional Carbon Trading with Additionality

Most of the institutions that evaluate and certify offsets generated by carbon-emissions reduction projects, such as The Climate Action Reserve (CAR) or The Voluntary Carbon Standard (VCS), only certify offsets that are additional to what would have occurred in the absence of the carbon market. Because Scheme I represents the business-as-usual scenario, we define the optimal harvesting policy obtained in Scheme I as our baseline. (See Murray et. al. 2007 for a discussion of baseline estimation approaches.) Thus, under Scheme IV, we only need to add the following constraint to the model for Scheme III:

$$u_t^{III}\left(x_t,P_t,P_t^C,z_t\right)\left(1-h_t^{I*}\left(x_t,P_t\right)\right)=0 \qquad (2.13)$$

This constraint states that, given $x_t$ and $P_t$, offsets can be sold at period $t$ only if the optimal harvesting policy under Scheme I for those values of $x_t$ and $P_t$ is to harvest at period $t$.

## 2.3 Stochastic Prices

Carbon and timber prices are two key variables in the optimization problem. While some previous studies (e.g. Clarke and Reed 1989) use non-stationary processes such as geometric Brownian motion (GBM) to model timber prices, we follow Haight and Holmes (1991) and Plantinga (2004), who model timber prices with a stationary process. In particular, we assume that the timber price follows a mean-reverting (MR) process. On the other hand, we assume that the carbon price evolves according to a GBM process. This assumption has been made by previous studies in the carbon price modeling literature (e.g. Chladná 2007 and Yang et al. 2008), and is consistent with the idea of expected rising carbon prices (e.g. Sohngen and Mendelsohn 2003).

The MR process for the timber price is given by $dP_t = \alpha\left(\mu_P - lnP_t\right)P_t dt + \sigma_P P_t dw_t$, where $\alpha$ is a mean-reversion coefficient, $\mu_P$ determines the long-term equilibrium price, $\sigma_P$ is the process volatility (standard deviation of the returns) and $dw$ is a Wiener process. (For a detailed definition and properties of the Wiener process see Luenberger 1998, chapter 11, or Ross 1996, chapter 8.) The GBM process that describe the evolution of the carbon price is given by $dP_t^C = \mu_{P^C} P_t^C dt + \sigma_{P^C} P_t^C dw_t^C$, where $\mu_{P^C}$ is the growth rate, or drift, $\sigma_{P^C}$ is the process volatility and $dw^C$ is a Wiener process. We assume that the increments $dw_t$ and $dw_t^C$ are correlated, with a correlation coefficient $\rho$. Furthermore, the corresponding Ito-transformed processes for $y_t = lnP_t$ and $y_t^C = lnP_t^C$

are $dy_t = \alpha\left(\bar{Y} - y_t\right)dt + \sigma_P dw_t$ and $dy_t^C = \nu_{y^C} dt + \sigma_{P^C} dw_t^C$, where $\nu_{y^C} = \mu_{P^C} - \sigma_{P^C}^2 / 2$ and $\bar{Y} = \mu_P - \sigma_P^2 / 2\alpha$ .

### 2.3.1 Finite-horizon Model

In order to find a numerical solution to the finite-horizon models formulated in section 2.2, we approximate the prices stochastic processes in discrete time using a two-dimensional binomial lattice (for a detailed description of binomial lattices see Luenberger 1998, chapter 11). To construct the lattice we need to calculate the magnitude of the up and down moves for each price. The up moves are given by $u_{P^C} = e^{(\sigma_{P^C})\sqrt{\Delta t}}$ and $u_{P^L} = e^{(\sigma_{P^L})\sqrt{\Delta t}}$, respectively, while the down moves are equal to the reciprocal of the up moves. Additionally, we need to calculate $\pi_{uu}$, $\pi_{ud}$, $\pi_{du}$, and $\pi_{dd}$, which represent the probabilities of (up, up), (up, down), (down, up) and (down, down) price moves, respectively.

If the price processes are assumed to be independent, the joint probabilities of prices moves are given by the product of the marginal probabilities, i.e., $\pi_{uu} = \pi_u \pi_u^C$, $\pi_{ud} = \pi_u \pi_d^C$, $\pi_{du} = \pi_d \pi_u^C$ and $\pi_{dd} = \pi_d \pi_d^C$, where

$$\pi_u^C = \frac{1}{2} + \Delta t \frac{\nu_{Y^C}}{2u_{Y^C}}$$

$$\pi_u = max\left\{0, min\left\{1, \frac{1}{2} + \Delta t \frac{\nu_Y}{2u_Y}\right\}\right\} \qquad (2.14)$$

If the price processes are correlated, we can follow the approach developed by Hahn and Dyer (2008, 2011) to determine the joint probabilities of price moves. The basic idea of this approach is to match the first two moments of the moves in the lattice with those from the continuous stochastic processes for the logarithm of the prices. However, depending on the degree of mean reversion required at some nodes in the lattice for the timber price, the approach can result in some probabilities being less than zero or greater than one. Because the joint probabilities of the moves cannot be directly censored, we calculate, and censor as necessary, the conditional probabilities of timber price moves given carbon price moves. Finally, we obtain the joint probabilities of the moves as the product of these conditional probabilities and the marginal probabilities of the carbon price moves. Thus, $\pi_{uu} = \pi_{u|u}\pi_u^C$, $\pi_{ud} = \pi_{u|d}\pi_d^C$, $\pi_{du} = \pi_{d|u}\pi_u^C$, and $\pi_{dd} = \pi_{d|d}\pi_d^C$, where the conditional probabilities are given by

$$\pi_{u|u} = max\left\{0, min\left\{1, \frac{u_{Y^C}\left(u_Y + \Delta t \nu_Y\right) + \Delta t\left(u_Y \nu_{Y^C} + \rho\sigma_{P^C}\sigma_P\right)}{2u_Y\left(u_{Y^C} + \Delta t \nu_{Y^C}\right)}\right\}\right\}$$

$$\pi_{d|d}^L = max\left\{0, min\left\{1, \frac{u_{Y^C}\left(u_{Y^L} - \Delta t \nu_{Y^L}\right) + \Delta t\left(\rho\sigma_{P^C}\sigma_{P^L} - u_{Y^L}\nu_{Y^C}\right)}{2u_{Y^L}\left(u_{Y^C} - \Delta t \nu_{Y^C}\right)}\right\}\right\} \quad (2.15)$$

### 2.3.2 Infinite-horizon Model

For the infinite-horizon models, we can calculate the transition probabilities by using the joint distribution of the prices $(y_t, y_t^C)$ given the initial prices $(y_0, y_0^C)$. Given our assumptions about the price processes, we can follow Schwartz and Smith (2000) to determine that $\left(y_t, y_t^C | y_0, y_0^C\right)$ has a bivariate normal distribution with mean vector $\left(\mu_{Y|Y_0} = \bar{Y} + \left(y_0 - \bar{Y}\right)e^{-\alpha t}, \mu_{y^C|y_0^C} = y_0^C + \nu_{y^C} t\right)$ and covariance matrix

$$\begin{pmatrix} \left(1-e^{-2\alpha t}\right)\dfrac{\sigma_P^2}{2\alpha} & \left(1-e^{-\alpha t}\right)\dfrac{\rho\sigma_P\sigma_{P^C}}{\alpha} \\ \left(1-e^{-\alpha t}\right)\dfrac{\rho\sigma_P\sigma_{P^C}}{\alpha} & \sigma_{P^C}^2 t \end{pmatrix}. \qquad (2.16)$$

We can then calculate the joint transition probabilities as the product of marginal and conditional probabilities. Using the properties of the normal distribution, we have that $y_t^C \left| y_0^C \right.$ is normally distributed with mean $\mu_{y^C|y_0^C}$ and variance $\sigma^2_{y^C|y_0^C} = \sigma^2_{P^C} t$, and that $y_t \left| y_t^C, y_0 \right.$ is also normally distributed with mean $\mu_{y|y_0} + \rho \dfrac{\sigma_{y|y_0}}{\sigma_{y^C|y_0^C}}\left(y_t^C - \mu_{y^C|y_0^C}\right)$ and variance $\left(1-\rho^2\right)\sigma^2_{y|y_0}$.

## 2.4 Results: Single-Rotation Finite-Horizon Problem

In this section we numerically solve the single-rotation finite-horizon problem and compare the optimal harvesting policies under the four carbon trading schemes for a hypothetical forest stand. We start the section by setting the values for the parameters of the model. Then, we describe and compare the optimal policies, optimal harvesting time and optimal reward under the different schemes. Finally, we discuss the impact of relaxing the mandatory “harvest and sell” assumption on the results.

## 2.4.1 Model Parameters and Price Scenarios

We define $T = 60$ as our time horizon and $\Delta t = 1$ (decisions are made once in every period). We also set $\gamma = 0.5\,tCO_2 / m^3$, $c = 0.4$, $r = 0.02$, and $\kappa = 1$. For the timber growth function, $f(x_t)$, we use the function developed by Birch (1999) and described in Chladná (2007):

$$f(x_t) = x_t(-0.0955 + 0.8121 x_t^{0.6823-1}) \qquad (2.17)$$

In order to decrease the computational requirements needed to numerically solve the DP models presented in section 2.2, we modify the growth function by making one period in our analysis equal to three periods of growth in Birch's function. Figure 1 shows how the timber volume changes over time given the growth curve used in our analysis.

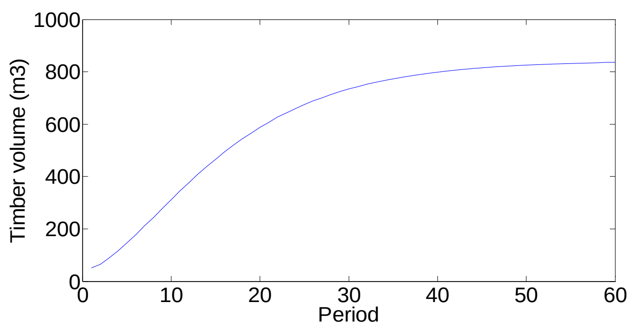



**Figure 1: Timber Volume.**

As we mentioned in the introduction, several papers have already studied the impact of timber price uncertainty on the optimal harvesting problem. For example, Plantinga (1998) and Chladná (2007) point out that modeling the timber price with a MR process leads to increases in the expected optimal rotation time relative to the optimal rotation time when the timber price is constant and equal to the long-term equilibrium level of the MR process. In our analysis, we use the values estimated by Chladná (2007) for the parameters of the timber price MR process: $P_0^L = 76\$ / m^3$, $\mu_{P^L} = 4.34$, and $\sigma_{P^L} = 0.09$. These values were estimated using historical data from 1994 to 2004 for the timber price in the Tirol, Austria, region.

Because our main goal in this work is to study the impact of carbon offset trading on the optimal harvesting policy, we focus our analysis on the parameters of the carbon price process. First, we set the initial carbon price, $P_0^C$, equal to $10\$ / tCO_2$. This value is roughly equivalent to the average forest offset price reported in the 2009 study by Ecosystem Marketplace. Second, we define $\sigma_{P^C} = 0.16$ and $\rho = 0.5$. These values were estimated by Chladná (2007) based on the carbon prices generated by the IIASA MESSAGE model (see Riahi and Roehrl 2001 for details). We did a sensitivity analysis on $\sigma_{P^C}$ and $\rho$ and found that changing these parameters has little impact on the optimal policy. Therefore, we treat the value of these two parameters as a constant for the remainder of the chapter.

Finally, we turn our attention to the carbon price drift, $\mu_{P^C}$, which will play a fundamental role in determining the optimal policy. According to Hotelling's rule (Hotelling 1931), in equilibrium, the carbon price expected growth should be equal to the forest manager's discount rate. However, in the short run, market equilibrium may not be achieved and, consequently, the forest manager's expected value for the carbon price growth may be different from her discount rate. Therefore, in our analysis, the carbon price drift may be higher or lower than the forest manager's discount rate.

### 2.4.2 Optimal Policy

In this section, we discuss some characteristics of the optimal harvesting and offset-selling policies, and the value function under the different offset-selling schemes.

*Scheme I*

Scheme I does not include offset trading, so the optimal policies and the value function are identical for different values of the carbon price drift. We first note that for a given period $t$ the value function, $V_t^I\left(x_t, P_t\right)$, is piecewise linear and increasing in $P_t$. For low timber prices, harvesting is not optimal, and the slopes are determined by the continuation values for each price. When timber price is high, harvesting is optimal and, therefore, the slope depends only on the harvesting cost and the current volume.

Regarding the structure of the optimal harvesting policy, $\{h_t^{I*}(x_t, P_t)\}_{t=1}^T$, we obtain a threshold policy, as shown in Figure 2 (a). In early periods, harvesting is never optimal because the forest is still growing at a fast rate. After some point, because $f\left(.\right)$ is concave, the timber growth rate slows down, and there is a threshold such that it is optimal to harvest if the timber price is above that threshold. To understand this, note that when $P_t$ is higher than the long term equilibrium level, it is expected to decrease in the future as a consequence of the MR process. Therefore, the forest manager may be better off by harvesting at period $t$.

Furthermore, Figure 2 (b) shows that the harvesting threshold (i.e. reservation price) generally decreases over time. (The curve is not smooth because of the discrete approximation used for the prices.) Note that this threshold policy with a decreasing reservation price is usually observed in optimal stopping problems with a deadline (e.g., Bertsekas 2007) and is consistent with the results observed in previous studies about the optimal rotation length with stationary timber prices (e.g. Plantinga 1998).

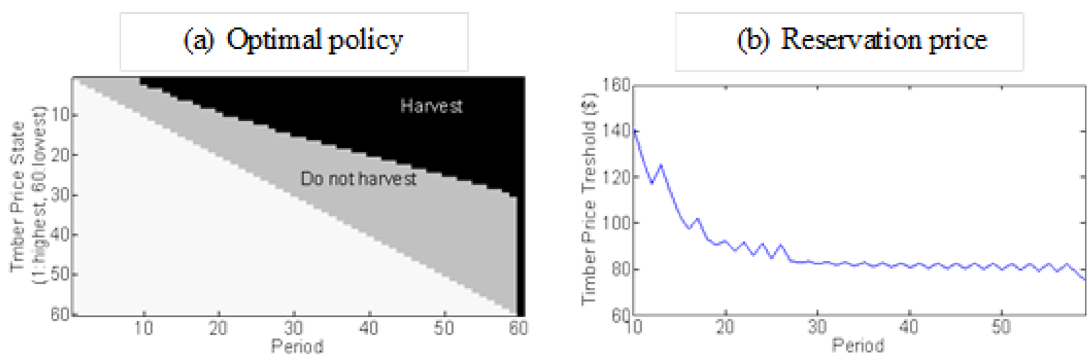



**Figure 2: Optimal Harvesting Policy under Scheme I.**

*Scheme II*

Under Scheme II, the carbon price becomes a relevant state variable. The value function, $V_t^{II}\left(x_t, P_t, P_t^C\right)$, is piecewise linear and increasing in $P_t$, while the relationship with $P_t^C$ depends on the harvesting penalty, $\kappa$, the parameters of the carbon price process, and the specific period $t$. For the case of $\kappa = 1$, when the carbon price expected growth per period is higher than the discount rate, the benefits from selling offsets in each period will not compensate for the high penalty that must be paid at the time of harvesting. This implies that $V_t^{II}\left(x_t, P_t, P_t^C\right)$ is decreasing in $P_t^C$ in every period.

In contrast, when the carbon price drift is lower than the discount rate, the value function may be either increasing or decreasing. In the early periods, the value function may be increasing in $P_t^C$ because the forest manager can benefit from selling offsets at a higher price during most periods, and this benefit will outweigh the increment on the harvesting penalty. However, if the number of periods in which the forest manager can sell offsets at a higher price is not large enough to compensate for the higher harvesting penalty, then the value function will be decreasing in $P_t^C$. Figure 3 illustrates this for periods $t=3$ and $t=30$ when $\mu_{P^C}=0.04$.

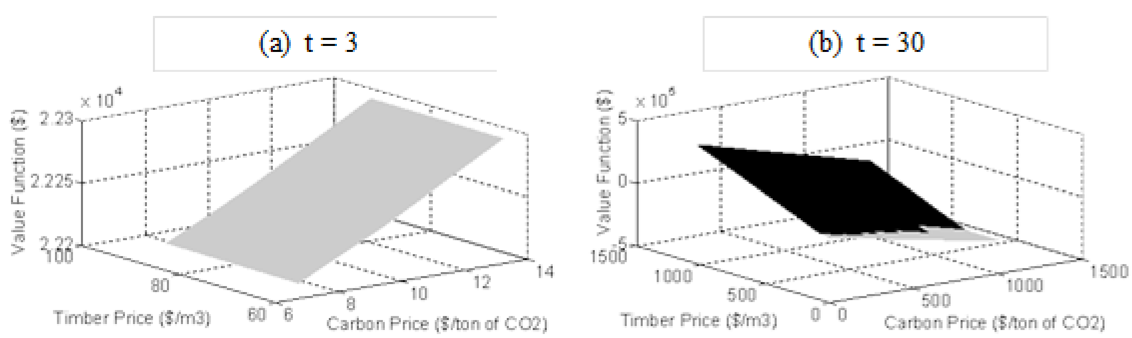



**Figure 3: Value Function under Scheme II.**

The structure of the optimal harvesting policy under Scheme II also depends on the parameters of the carbon price process. Figure 4(a) and Figure 4(b) show the optimal harvesting policy at period 30 for the cases $\mu_{p^C} = 0.04$ and $\mu_{p^C} = 0.01$, respectively. As in Scheme I, we can see that there is an upper threshold such that it is optimal to harvest if the timber price is above that threshold. However, in this case the carbon price also plays a role in the harvesting decision.

When the carbon price drift is higher than the discount rate (e.g. $\mu_{p^C} = 0.04$) if the carbon price is high enough, we have a lower threshold such that it is optimal to harvest if the timber price is below that threshold, even though the timber price may be expected to increase. In this case the forest manager prefers to harvest in order to avoid a higher harvesting penalty in the future; because the carbon price follows a GBM process, it is expected to increase in each period at a rate higher than the discount rate. Furthermore, note that the upper threshold is non-increasing in the carbon price while the lower threshold is non-decreasing in the carbon price.

When the carbon price drift is lower than the discount rate (e.g. $\mu_{p^C} = 0.01$) we do not have a lower threshold, and the upper threshold is non-decreasing in the carbon price. To understand this, note that because the carbon price's expected growth rate is smaller than the discount rate, the marginal benefit from the subsidy outweighs the marginal harvesting penalty, implying that the forest manager has an incentive to delay harvesting. Therefore the higher the carbon price, the higher the timber price must be for harvesting to be the optimal decision.

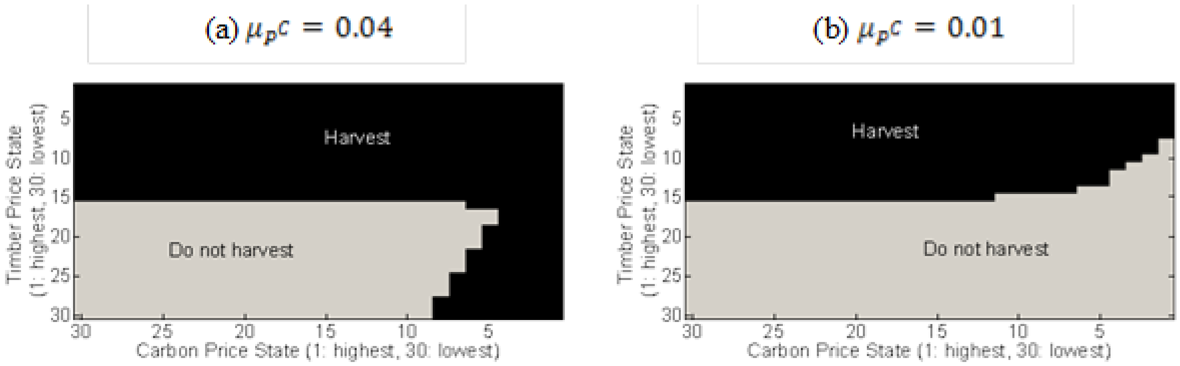



**Figure 4: Optimal Harvesting Policy under Scheme II at *t*=30.**

*Schemes III and IV*

Under schemes III and IV, in addition to the harvesting decision, the forest manager also has to decide in each period whether or not to sell carbon offsets. This decision will depend on the carbon price process as well as on the harvesting penalty and the discount rate. First, note that if the carbon price expected growth is equal to the forest manager's discount rate (this would be the case if we apply Hotelling's rule), then the forest manager would be indifferent between selling or not selling offsets, because the expected future value of the benefits from the offsets would be equal to the expected reversal penalty that must be paid at the time of harvesting.

When the expected growth rate of the carbon price is higher than the discount rate, the penalty that must be paid for every offset sold will be higher than the future value of the money obtained from selling that offset. Therefore, not selling offsets at any period is optimal for the forest manager under schemes III and IV. As a consequence, the optimal harvesting policy and the value function under schemes III and IV will be identical to those under Scheme I.

However, if the carbon price drift is lower than the discount rate, the future value of the money obtained from selling an offset will be higher than the penalty that must be paid for that offset at the time of harvesting. Therefore, selling offsets in each period is optimal, and the optimal harvesting policy and the value function under Scheme III will be similar to that under Scheme II. Given the additionality requirement included in Scheme IV, the value function will be lower under this scheme. However, because selling fewer offsets is still better than selling no offsets, the optimal harvesting policy under Scheme IV is the same as under Scheme III.

### 2.4.3 Optimal Harvesting Time and Optimal Reward

In this section we focus on the analysis of two relevant variables: the optimal harvesting time, $t^*$, and the optimal reward, $V^*$. While $V^*$ is the most important variable for the forest manager, $t^*$ is a variable of special interest for a social planner interested in increasing the average carbon sequestration. We calculate the expected value of $t^*$, $Et^*$, by simulating 5000 paths for the timber and carbon prices in the binomial lattice. We then use the optimal policies described in section 2.4.2 to calculate $t^*$ for each path and, finally, we take the average of $t^*$ over all paths. The optimal reward, $V^*$, is equal to the value function at time 0. Figure 5 shows how $Et^*$ and $V^*$

change for different values of $\mu_{p^C}$ , while keeping the other parameters of the price processes at their original values.

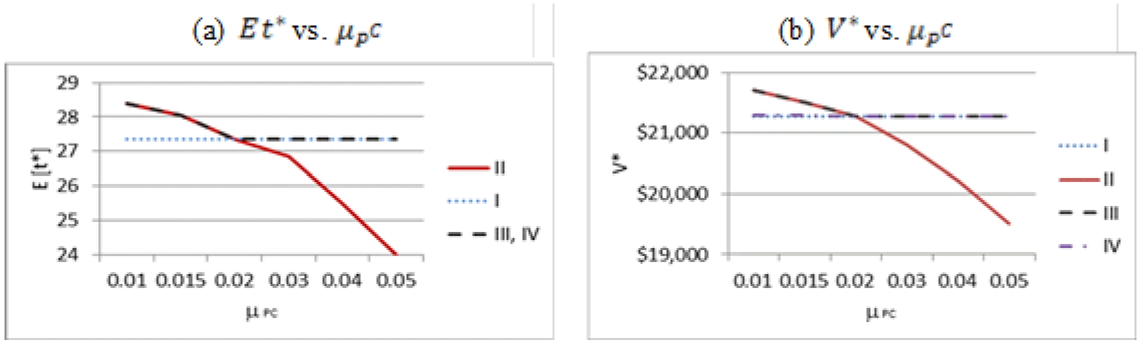



**Figure 5: Optimal Expected Harvesting Time and Optimal Reward vs. Carbon Price Drift.**

As we mentioned in section 2.4.2, when the carbon price process has a drift higher than the discount rate, the forest manager never sells offsets under schemes III and IV. Therefore, the optimal harvesting policy and, consequently, $Et^*$ and $V^*$ under schemes III and IV are identical to those under Scheme I. Scheme II, however, does not give the forest manager the option of deciding whether to sell offsets. In this case, as Figure 4(a) in section 2.4.2 shows, harvesting is optimal not only when the timber price is higher than an upper-threshold but also when a low timber price combines with a high carbon price. This increases the probability of an early harvest with a low profit, which leads to decreases in both $Et^*$ and $V^*$ relative to the other three schemes, as Figure 5 shows.

When the carbon price process has a drift lower than the discount rate, the forest manager sells offsets in every period (when possible) under schemes III and IV.

Therefore, $Et^*$ under schemes III and IV are identical to those under Scheme II. $V^*$ is lower under Scheme IV than under schemes II and III because of the additionality constraint. Additionally, the fact that the harvesting threshold under Scheme II is non-decreasing in the carbon price (as Figure 4(b) shows) implies that the forest manager has an incentive to delay harvesting. Consequently, $Et^*$ and $V^*$ are higher under schemes II, III and IV than those under Scheme I.

### 2.4.4 Risk Aversion and Jump Price Process

In this section we briefly analyze two aspects that may have an impact on the optimal harvesting policy: the forest manager's risk aversion and the possibility of a jump in the carbon price process. (The detailed results for this section are available upon request.)

So far, we have assumed that the forest manager is risk neutral. However, assuming that the forest manager is risk averse may makes sense if, for example, the market is not at equilibrium, and the forest manager is in charge of few forest stands. To model risk aversion, we assume that the forest manager has an exponential utility function and wants to maximize the expected utility of the NPV of the income cash flow. (See e.g. Bertsekas 2007 for the DP algorithm corresponding to an exponential cost function.) Similar to Clarke and Reed (1989), we found that incorporating risk aversion leads to a decrease in $Et^*$ relative to the risk neutral case. This effect is stronger when the level of risk aversion increases. However, in our particular example, the decrement in $Et^*$ is small. For example, with a risk tolerance parameter equal to 30,000, $Et^*$ decreases between 0.3 and 0.5 periods in the different schemes.

Finally, in addition to the stochastic variation of the carbon price, captured in the GBM process, we also studied the possibility of a positive jump in the carbon price. This may occur as a product of a special event such as a significant change in the carbon emissions regulation. As in the no-jump case, flexibility of schemes III and IV allows the forest manager to better respond to possible price changes. On the one hand, if the jump is expected to occur during the early periods, the forest manager will generally start selling offsets (if the carbon price drift is lower than the discount rate) only after the jump, and $Et^*$ will usually be higher than in the no-jump case. On the other hand, if the jump is expected to occur at intermediate or late periods, the forest manager may either not sell any offsets, or start selling offsets from the beginning but stop selling these earlier, because of the risk that a jump occurs before harvesting. In this case, $Et^*$ generally decreases with respect to the no-jump case.

### 2.4.5 Relaxing the Mandatory "Harvest and Sell" Assumption

One of the assumptions that we made in our previous analysis is that the forest manager must harvest and sell the land at period $T$ or earlier. If, instead, the forest manager has the option of never harvesting and therefore never paying a harvesting tax, then some of our previous results will change. First, consider the case where the carbon price drift is lower than the discount rate. Because in this case the harvesting tax (or similarly, the reversal penalty) is lower than the value of the subsidies received (or similarly, the offsets sold), the forest manager generally prefers to harvest at some point in time and thereby obtain the timber benefits (unless the timber prices are very low). Therefore, in our numerical example, the harvesting and offset-selling optimal policies as well as the optimal rotation length and the optimal reward are similar to those obtained under the “harvest and sell” assumption.

If the carbon price drift is higher than the discount rate, the harvesting tax is higher than the value of the subsidies received. Therefore, in some cases where the carbon price is too high, never harvesting is the forest manager’s optimal decision. Figure 6(a) shows the optimal harvesting policy under Scheme II, when “never harvesting” is possible. Compared to Figure 4(a), Figure 6 does not have a lower-threshold. This means that in cases with a high carbon price and a low timber price, the forest manager does not harvest. Furthermore, in these cases she will never harvest because the carbon price, and therefore the harvesting tax, are expected to increase even more.

The optimal harvesting and offset-selling policies under schemes III and IV also change. Remember that under the “harvest and sell” assumption, the optimal harvesting policy under schemes III and IV is identical to that under Scheme I. In contrast, when “never harvesting” is possible, the forest manager has the option of selling offsets and never harvesting. Therefore, the higher the carbon price (and therefore the benefits from selling offsets), the higher the timber price needed to make harvesting optimal, as we can see in Figure 6(b). Finally, note that because the carbon price is expected to increase at a high rate in every period, when the forest manager decides to sell offsets and never harvest, she will wait as long as she can in order to sell the offsets at a higher price. (The period in which she sells the offsets will depend on the constraint (if any) regarding the maximum age of the stand at which offsets can be sold.)

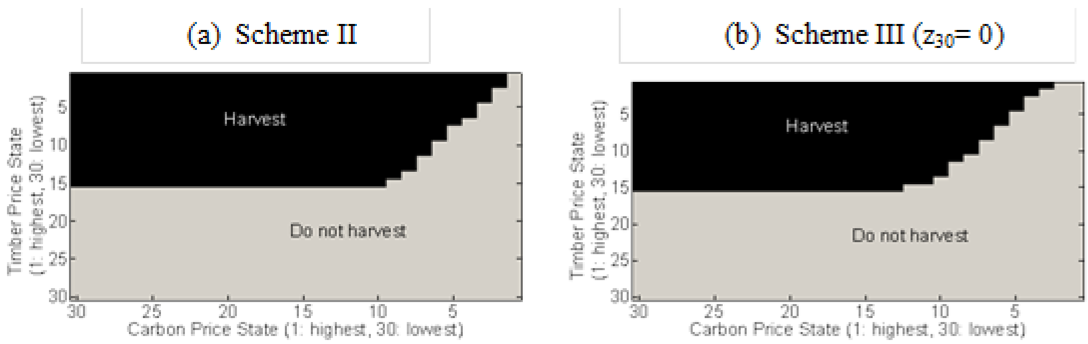



**Figure 6: Optimal Harvesting Policies under the "Never Harvest" Option at *t*=30.**

As a consequence of the changes on the optimal policies when "never harvesting" is possible, $Et^*$ and $V^*$ also change. As we previously mentioned, never harvesting may be optimal in some cases. Because an expected value of optimal harvesting cannot be calculated in this case, we can instead look at the histogram of the harvesting time (Figure 7). Figure 7(a) shows that under Scheme II, never harvesting is optimal about 3% of the time. This percentage increases to about 14% under Scheme III (Figure 7(b)), because the forest manager can wait and sell the offsets at a higher price at the last possible period and, therefore, the benefits from never harvesting and selling the offsets are higher. (Here we assume that $T = 60$ is the deadline to sell the offsets.)

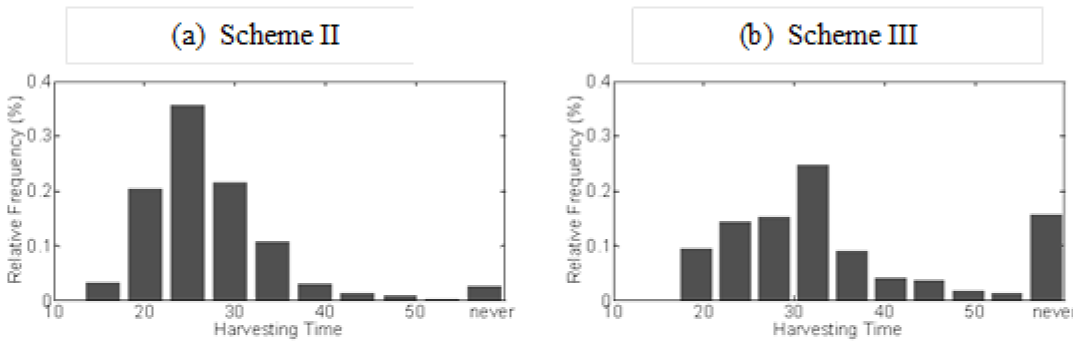



**Figure 7: Optimal Harvesting Time under the "Never Harvest" Option.**

Finally, note that if the forest manager is required to pay a reversal penalty at some point, even if she never harvest, the results would be very similar to those obtained in the previous sections under the "harvest and sell" assumption.

## *2.5 Results: Multiple-rotations Infinite-Horizon Model*

In section 2.4 we studied the single-rotation finite-horizon forest harvesting problem, under the assumption that the forest manager must harvest at period $T$ or earlier. While this assumption may be reasonable in some cases, a more realistic and general setting would allow the forest manager to replant the forest after every harvest and start a new rotation. In this section, we present the results of the multiple-rotation infinite-horizon forest harvesting problem. As in the previous section, we first characterize the optimal harvesting policy in the case of no carbon offsets trading, and then analyze the impact of carbon offsets on the optimal harvesting policy.

In contrast to the previous section where we presented only numerical results, in this section we provide both analytical and numerical results to illustrate some properties of the value function and the optimal harvesting policy under the different schemes. For the numerical analysis of the discrete-time infinite-horizon dynamic programming problems, we discretize the state space (60 states for the volume, 20 states each for the timber and carbon prices, for a total of 24000 states) as well as time (into intervals of 1 year). The dynamic program was formulated as a linear program (as in Bertsekas 2007, p.416) and solved using the MOSEK toolbox for MATLAB.

*Scheme I*

We now characterize the value function and the optimal harvesting policy under Scheme I. All proofs are presented in the Appendix.

*PROPOSITION 2.1. Assume that the reward function per stage* $r(x, y, h)$ *is bounded. Then, the value function has the following properties:*

1. $J^*(x, y)$ *is non-decreasing and convex in* $y$*, for any* $x$*.*
2. $J^*(x, y)$ *is non-decreasing in* $x$*, for any* $y$*.*
3. $J(x, y, h)$ *is supermodular in* $(y, h)$*, for any* $x$*.*

The first two results of Proposition 2.1 show that, under the optimal policy, the forest manager's payoff increases with both the timber volume and the timber price. This intuitive finding is illustrated in Figure 8. Additionally, the optimal value function is convex in the log of the timber price. This is a consequence of the convexity of the reward function $r(x, y, h)$ as well as the stochastic convexity of the price transitions. Note that this condition implies that increasing the uncertainty in the timber price increases the value function and, therefore, benefits the forest manager.

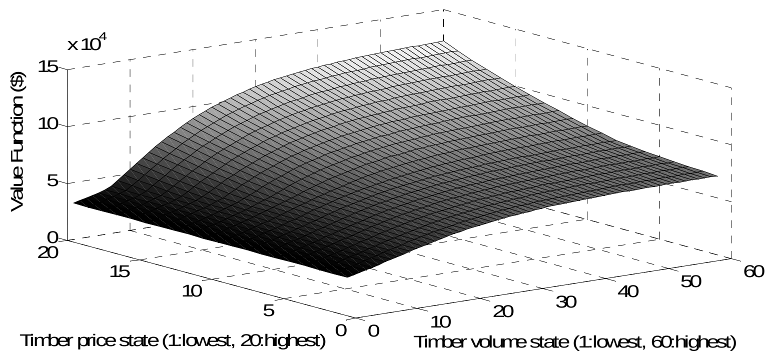



**Figure 8: Value Function under Scheme I (Infinite-horizon).**

The third result of Proposition 2.1 states that the value function is supermodular in $(y, h)$. This means that, for a given volume, the incremental gain of harvesting is greater when the timber price is higher or, equivalently, that the difference of the value functions $\nabla(x, y) = J(x, y, 1) - J(x, y, 0)$ is increasing in $y$. This result will be particularly useful to characterize the optimal harvesting policy, which we do next.

*PROPOSITION 2.2. Assume that the reward function per stage $r(x, y, h)$ is bounded. Then, the optimal harvesting policy has the following properties:*

1. *For a given volume, the optimal harvesting policy is a threshold policy in the timber price: if it is optimal to harvest at $(x, y)$, it is also optimal to harvest at $(x, y')$ for any $y' \geq y$.*

2. *Define* $\overline{x}$ *as the minimum* $x$ *satisfying* $f'(x) \le 1/\delta E\left[e^{\tilde{z}}\right]$*. For a given timber price, the optimal harvesting policy is a threshold policy in the timber volume, for* $x \ge \overline{x}$ *: if it is optimal to harvest at* $(x, y)$*, it is also optimal to harvest at* $(x', y)$ *for any* $x' \ge x$.

Figure 9 illustrates the results of Proposition 2.2. In early periods, harvesting is generally not optimal because the forest is still growing at a fast rate. After some point, because $f(x)$ is concave, the timber growth rate slows down and there is a threshold such that it is optimal to harvest if the timber price is above this threshold.

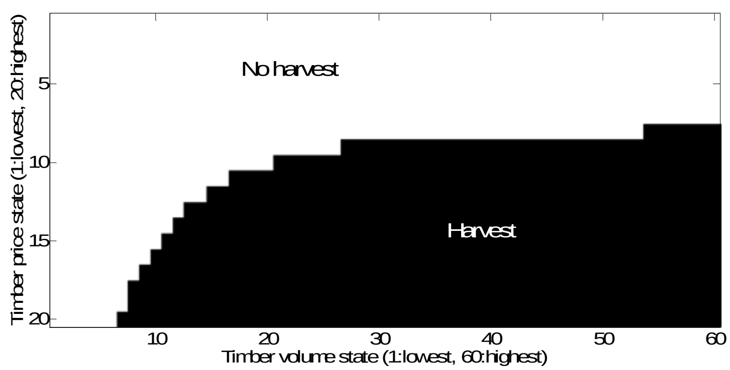



**Figure 9: Optimal Harvesting Policy under Scheme I (Infinite-horizon).**

To better understand the second result, note that $f'(x)$ represents the timber growth rate while $\delta E\left[e^{\tilde{z}}\right]$ represents the discounted expected change in the timber price. Thus, when any of these two expressions increase, the value function of waiting becomes more attractive relative to the value function of harvesting. Specifically, when $f'(x) > 1/\delta E\left[e^{\tilde{z}}\right]$, the difference of the value functions $\nabla(x, y) = J(x, y, 1) - J(x, y, 0)$ may be decreasing in $x$ and, therefore, we cannot guarantee a threshold policy in $x$.

Note that the condition $f'(x) \le 1/\delta E\left[e^{\tilde{z}}\right]$ is less likely to hold when the timber volume is low because $f'(x)$ is decreasing. Once the condition holds for a given $x$, it will hold as well for any $x' \ge x$. Finally, note that Proposition 2 implies that the thresholds characterizing the optimal policy are non-increasing. As we can observe in Figure 9, the timber price required to harvest the forest decreases as the timber volume increases and, similarly, the timber volume required to harvest decreases as the timber price increases.

*Scheme II*

Under Scheme II, the carbon price becomes a relevant state variable. Before analyzing the impact of the carbon price on the value function and the optimal harvesting policy, we first note that most of the properties related to the timber volume and the timber price discussed for Scheme I will also hold for Scheme II. These results are summarized in Propositions 2.3 and 2.4.

*PROPOSITION 2.3. Assume that the reward function per stage* $r\left(x, y, y^c, h\right)$ is *bounded. Then, the value function has the following properties:*

1. $J^*\left(x, y, y^c\right)$ *is non-decreasing and convex in* $y$*, for any* $(x, y^c)$.
2. *If* $f'(x) \ge 1$ *and* $f'(x) \ge \gamma e^{y^c} / (e^y + (1-\kappa)\gamma e^{y^c})$*, then* $J^*\left(x, y, y^c\right)$ *is non-decreasing in* $x$*, for any* $(y, y^c)$.
3. $J\left(x, y, y^c, h\right)$ *is supermodular in* $\left(y, h\right)$*, for any* $(x, y^c)$.

Note that in this case, the value function is non-decreasing in $x$ only when the marginal timber growth is higher than a lower bound, which is more likely to happen for low timber volumes.

*PROPOSITION 2.4. Assume that the reward function per stage* $r\left(x, y, y^c, h\right)$ *is bounded. Then, the optimal harvesting policy has the following properties:*

1. *The optimal harvesting policy is a threshold policy in the timber price* $y$ *: if it is optimal to harvest at* $\left(x, y, y^c\right)$*, it is also optimal to harvest at* $\left(x, y', y^c\right)$ *for any* $y' \geq y$ *.*

2. *Define* $\overline{x}$ *as the minimum* $x$ *satisfying* $f'\left(x\right) \leq \frac{e^{y} + \delta E\left[e^{\tilde{w}}\right]\gamma e^{y^c} - \kappa\gamma e^{y^c}}{\delta E\left[e^{\tilde{z}}\right]e^{y} + \delta E\left[e^{\tilde{w}}\right]\gamma e^{y^c} - \kappa\delta E\left[e^{\tilde{w}}\right]\gamma e^{y^c}}$ *. If* $e^{y} - \kappa\gamma e^{y^c} \geq 0$*, the optimal harvesting policy is a threshold policy in the timber volume* $x$ *, for* $x \geq \overline{x}$ *: if it is optimal to harvest at* $\left(x, y, y^c\right)$*, it is also optimal to harvest at* $\left(x', y, y^c\right)$ *for any* $x' \geq x$ *.*

Note that, similar to the case of Scheme I, the optimal policy is a threshold policy in the timber volume when a particular condition on $f'\left(x\right)$ is satisfied. In this case, the condition on $f'\left(x\right)$ considers the current and the expected future prices for both timber and carbon.

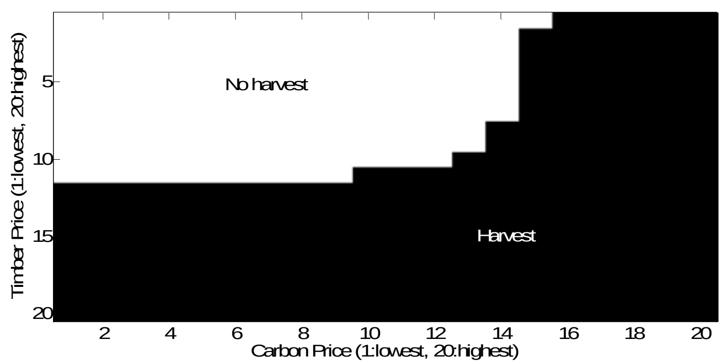
Obtaining analytical results to illustrate the impact of the carbon price on the optimal harvesting policy is more complicated. First, note that the forest manager will receive payments for the carbon sequestered at each period but then she will have to pay a penalty for the carbon released at the time of harvesting. Therefore, the reward function $J\left(x, y, y^c, h\right)$ is not necessarily supermodular in $\left(y^c, h\right)$ and the approach used to study the impact of the timber price on the optimal policy cannot be used. Additionally, as opposed to the timber volume, the carbon price has an impact on the value to go function not only when the forest manager decides to wait ($J^*\left(f\left(x\right), \tilde{g}\left(y\right), \tilde{h}\left(y^c\right)\right)$) but also when she decides to harvest ($J^*\left(x_0, \tilde{g}\left(y\right), \tilde{h}\left(y^c\right)\right)$).

To study the impact that the carbon price may have on the optimal harvesting policy, we can study the difference of the value functions, $\nabla\left(x, y, y^c\right) = J\left(x, y, y^c, 0\right) - J\left(x, y, y^c, 1\right)$. If this difference function were increasing in $y^c$ for given values of $x$ and $y$, we would have a threshold policy in $y^c$ such that if it is optimal to wait at $\left(x, y, y^c\right)$, it is also optimal to wait at $\left(x, y, y^{c}{}'\right)$ for any $y^{c}{}' \geq y^c$. If we use the value iteration method, we can write the difference function at the $k+1$ iteration as

$$\nabla_{k+1}\left(x, y, y^c\right) = \delta E\left[J_k\left(f\left(x\right), \tilde{g}\left(y\right), \tilde{h}\left(y^c\right)\right)\right] - e^y x + R + \kappa e^{y^c} \gamma x +$$

$\delta E\left[J_k\left(x_0, \tilde{g}\left(y\right), \tilde{h}\left(y^c\right)\right)\right]$, which can be shown to be either increasing or decreasing in $y^c$, depending on the timber volume, the harvesting penalty, and the parameters of the carbon price process. To illustrate this interaction, we can show that if postponing harvesting is optimal in the first $k$ iterations of the value iteration method, then, for the case of a reversal penalty of one, $\nabla_{k+1}\left(x, y, y^c\right)$ is increasing in $y^c$ if the discounted expected carbon price change $\delta E\left[e^{\tilde{w}}\right]$ is smaller than one or, equivalently, if the drift of the carbon price process (not the log of the price), $\mu_{p^C}$, is smaller than the discount rate $r$. Thus, in this case we would have a threshold policy on the carbon price, as illustrated in Figure 10b.

(a) $\kappa = 1, r = 0.01, \mu_{p^C} = 0.04$, timber state #20




(b) $\kappa = 1, r = 0.06, \mu_{p^C} = 0.04$, timber state #20

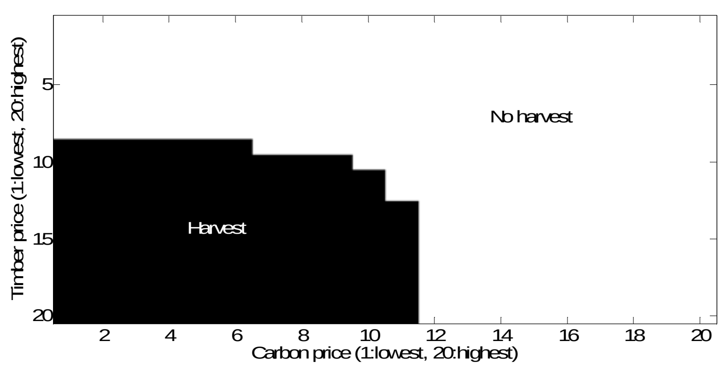



**Figure 10: Optimal Harvesting Policy under Scheme II (Infinite-horizon).**

While it is hard to obtain analytical results for general values of the penalty $\kappa$, the discount rate $r$, and the carbon price drift $\mu_{p^C}$, we can analyze some numerical results to understand the impact of these parameters on the optimal harvesting policy. For a given combination of timber volume, timber price, and harvesting penalty, the forest manager faces a tradeoff: if the increment in the future carbon payments obtained by harvesting the forest and starting from $x_0$ outweighs the harvesting penalty, the forest manager will harvest the forest; otherwise, she will postpone harvesting. Note that the first case is more likely to occur when the carbon price drift is higher relative to the discount rate. In this case, the function $\nabla\left(x, y, y^c\right)$ is decreasing in $y^c$, thus if harvesting is optimal at $\left(x, y, y^c\right)$, it will also be optimal at $\left(x, y, y^{c}{'}\right)$ for any $y^{c'} > y^c$, as shown in Figure 10a. In the second case, $\nabla\left(x, y, y^c\right)$ is increasing in $y^c$, thus if waiting is optimal at $\left(x, y, y^c\right)$, it will also be optimal at $\left(x, y, y^{c}{'}\right)$ for any $y^{c'} > y^c$, as shown in Figure 10b.

Finally, note that a harvesting policy with a structure like the one shown in Figure 10a, will lead to earlier expected harvesting compared to the optimal policy in Scheme I because the forest manager will harvest at some timber prices in which she would not harvest in Scheme I. On the contrary, a harvesting policy with a structure like the one in Figure 10b, will lead to a delay in the expected harvesting time. Therefore, a social planner interested in increasing carbon sequestration by delaying harvesting, should strategically choose a harvesting penalty that achieves this goal.

*Scheme III*

Under Scheme III, in addition to the harvesting decision, the forest manager also has to decide whether or not to sell carbon offsets in each period. This decision will depend on the carbon price drift as well as on the harvesting penalty and the discount rate. Similar to the results in the previous section, if the discount rate is higher than the carbon price drift, then selling offsets at each period is optimal, and the harvesting policy will be similar to that on Scheme II. However, if the discount rate is lower than the carbon price drift, the forest manager will wait to sell the offsets just before harvesting, in the case that the reversal penalty is lower than one, or until the carbon price achieves a maximum level, provided that an upper bound exists on this price.

## 2.6 Summary and Conclusions

In this chapter, we used dynamic programming to find the optimal harvesting and offset-trading policies for an even-aged forest stand that sells timber and carbon offsets under different schemes. In addition to the subsidies-tax scheme, used by previous papers in the optimal harvesting literature to incorporate carbon sequestration benefits, we also studied an "optional-selling" scheme where the forest manager has the option (but not the obligation) of selling offsets in each period. Furthermore, we incorporated into our model the additionality constraint required by most offset-certifying organizations.

We first study the single-rotation finite-horizon harvesting problem, where the forest manager must harvest the forest before a given time period. As a first general observation, we note that given our assumptions about the price processes, selling the timber at the appropriate time is more important to the forest manager than the offset-selling strategy. This is understandable, because the impact of selling timber is generally higher than the impact of selling offsets on the reward function. However, as the carbon price increases the impact of the offset-selling strategy on the optimal harvesting policy and the optimal reward will also increase. Both the magnitude and the direction of this impact will depend on the specific carbon scheme under which the forest operates.

Under a subsidies-tax scheme (Scheme II), incorporating carbon sequestration into the optimal harvesting problem may benefit or hurt the forest manager, depending on the carbon price behavior. If the carbon price is expected to be high and stable, the forest manager will generally delay harvesting to obtain additional benefits from carbon sequestration. However, if the carbon price is expected to increase over time, either with a sustained growth or with a jump at a specific period, the tax that must be paid by the forest manager at the time of harvesting will generally overweigh the subsidies received in the previous periods. Therefore, the forest manager will be worse off compared to the case where carbon sequestration is not considered (Scheme I).

When trading offsets is optional (Scheme III), incorporating carbon sequestration never hurts the forest manager. If selling offsets in every period is beneficial to the forest manager (such as in the case of high constant carbon prices or the case where the carbon drift is lower than the forest manager's discount rate), the optimal harvesting policy and the optimal reward are similar to those obtained under Scheme II. However, if selling offsets in every period does not benefit the forest manager, then Scheme III gives her the possibility of selling them only in some periods. For example, if the carbon price is expected to increase in every period at a high rate, the reversal penalty will generally overweigh the profit from the offsets sold in the previous periods, so the forest manager will not sell any offsets. In another example, if the forest manager expects a significant jump in the carbon price at a particular period, she may opt for selling offsets just after the jump occurs.

When the additionality constraint is incorporated (Scheme IV), the forest manager will be able to sell offsets only in the cases where the harvesting is delayed with respect to the business-as-usual management. As a consequence, the forest manager's optimal reward will decrease with respect to Scheme III but will still be better or equal to that obtained under Scheme I. Regarding the harvesting time, under the assumption that the penalty for offsets reversal is constant and for the price scenarios considered in this chapter, the additionality constraint does not affect the optimal harvesting time. However, under some other price assumptions (such as non-constant drifts) or some other penalty structure, there may be cases where the forest manager finds optimal to delay harvesting under Scheme III but not under Scheme IV.

Finally, we extend our analysis to the multiple-rotations infinite-horizon problem, where the forest manager replants the forest after every harvest. We characterize the value function under the different schemes and show that, once the forest achieves a minimum volume, the optimal harvesting policy is a threshold policy in the timber price as well as in the timber volume. Similar to the single-rotation problem, the impact of the carbon price on the optimal harvesting policy depends on our assumptions about the penalty $\kappa$, the discount rate $r$, and the carbon price drift $\mu_{p^C}$. The combination of high carbon prices and a high carbon price drift (relative to the discount rate) increases the expected value of the future carbon payments that can be obtained when the forest is in its initial growing stages. This increase in carbon payments may outweigh the harvesting penalty, leading the forest manager to harvest at some timber prices in which she would not harvest in the absence of offsets trading. Note that even though early harvesting is possible in both the finite-horizon and the infinite-horizon problems, the motivation for this is different. While in the finite-horizon problem early harvesting may occur because of the potential harvesting penalty that must be paid, in the infinite-horizon problem it may be caused by the opportunity of receiving higher carbon payments in the future.

### 2.6.1 Insights for Social Planers

Although most of the analysis in this chapter has been done from the forest manager's perspective, we can use some of the results to obtain insights for social planners interested in increasing forest carbon sequestration (i.e. extending the rotation length). From this perspective, schemes with optional offset trading (schemes III and IV) are clearly better than a subsidies-tax scheme (such as Scheme II), because the expected optimal rotation length is never lower in the former compared to the later. Particularly, if the carbon price has a strong increasing trend and the forest manager must "harvest and sell" before a given period, the optimal rotation length can be significantly lower under Scheme II than in the optional-trading schemes.

The effectiveness of an optional offset-selling scheme will depend on the specific conditions of the offset-selling agreements. In our setting, there are two key variables in these agreements that a social planer should carefully define: the length of the contract and the reversal penalty structure. In this chapter we have assumed that the offset-selling agreements have an infinite length. In some cases this may lead to an infinite rotation length, but in some other cases, it makes the mechanism ineffective, because the forest manager does not sell any offsets. Neither of these two outcomes is optimal to maximize carbon sequestration. In the first case, the trees will start to die after some point, releasing most of the carbon sequestered back to the atmosphere (a small portion of the carbon would remain sequestered in trees' residues and forest soil). In the second case, the rotation length does not change with respect to the business-as-usual scenario. Therefore, a social planer would choose a contract length such that the forest manager is willing to sell the offsets and delay harvesting with respect to the business-as-usual operation.

Another relevant variable in the offset-selling agreements is the reversal penalty. In our analysis we have assumed that this penalty is constant over time. Alternatively, a social planer could impose a reversal penalty that changes with the age of the offsets reversed. In this way, reversal of "younger" offsets (i.e. those recently sold) would be charged higher penalties. In this case, the forest manager will have more incentive to sell offsets in the early periods and less incentive to do so in the periods close to harvesting.

Incorporating the length of the contracts and the structure of the reversal penalty into our DP formulation would be somewhat complicated because in each period, in addition to the total number of offsets sold in the previous periods, we would also need information about the age of these offsets, increasing the state space in a considerable way. Nevertheless, the use of approximate dynamic programming tools to solve this problem would be a topic of interest for future studies.

# 3. Outsourcing Sustainability Efforts

## *3.1 Motivation and Literature Review*

On January 25th of 2012, The New York Times published a detailed report[1] describing the poor working conditions in a Chinese factory belonging to Foxconn, one of Apple's largest suppliers. On February 9th of 2012, CNNMoney[2] reported that about 250,000 people asked Apple, both online and at Apple stores, to improve working conditions in its suppliers' factories. The same month, Apple partnered with the Fair Labor Association (FLA) to conduct a voluntary audit about working and living conditions in three of the Foxconn factories in China. The results of the investigation, released on March 29th of 2012 and widely covered by US media, highlighted "at least 50 issues related to the FLA Code and Chinese labor law, including in the following areas: health and safety, worker integration and communication, and wages and working hours" (Fair Labor Association 2012). FLA's report also describes the main points of a remediation plan prepared by Apple and Foxconn to address each of the issues identified.

Labor conditions are just one of the several aspects included in the concept of sustainability, an approach that considers environmental and social impacts as pillars for long-term success. The Apple-Foxconn example illustrates the importance of

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[1] http://www.nytimes.com/2012/01/26/business/ieconomy-apples-ipad-and-the-human-costs-for-workers-in-china.html (accessed April 5, 2012).
[2] http://money.cnn.com/2012/02/09/technology/apple_foxconn_petition/ (accessed April 5, 2012).

sustainability performance for different parties. First, consumers' protests and wide media coverage showed that Apple's stakeholders were concern about the issues identified. Second, Apple's quick reaction to the problem, partnering with the FLA to conduct voluntary audits to the factories and designing a remediation plan to address the identified issues, demonstrates its interest in the sustainability practices of its suppliers. Finally, Foxconn's commitment to implement the remediation actions is a clear sign of its willingness to improve its sustainability performance.

Walmart, GE, Nike, and Adidas are other examples of large companies working to improve their sustainability performance (Plambeck et. al. 2012). Many of these strategies are motivated by increasing stakeholder pressure on the companies to implement sustainability initiatives. The World Business Council for Sustainable Development (2008) reported that consumer awareness and willingness to act on environmental concerns rose in most countries over the time period studied. In the US, for example, it increased from 57% in 2007 to 80% in 2008. As a response to this increasing pressure, many companies are changing their approaches to sustainability, from isolated social and environmental projects to corporate sustainability strategies and practices that are part of the core business. Goals, strategies, actions, and results on sustainability performance are often published on companies' websites.

Not only companies but also the governments and communities may be interested in leading sustainability initiatives. For example, several municipalities in

North Carolina work with Clean Energy Durham, a non-profit organization, to help community members learn and implement energy-saving techniques. The program, known as Pete Street™, consist of manuals, training programs, and consulting that municipalities, counties, utilities, or a coalition of community agencies can use to launch and run energy saving initiatives[3].

Organizations interested in implementing initiatives to improve their sustainability performance may not always have the capacity or control required to implement these initiatives directly. For example, Apple does not have full control over the operation of its suppliers and, therefore, cannot directly implement initiatives to improve their sustainability performance. However, Apple can support the implementation of these initiatives by providing incentives and support to their suppliers. Similarly, neighborhoods may not have the technical knowledge to develop and implement energy saving programs, but they can contract with Clean Energy Durham to help them accomplish their goals.

In this chapter, we develop a framework to study the role of organizations in the implementation of sustainability initiatives. We consider a buyer (e.g. Apple or a North Carolina municipality) who outsources sustainability efforts (e.g. improving working conditions or implementing energy saving programs) to a seller (e.g. Foxconn or Clean Energy Durham). Buyer's stakeholders (e.g. customers, financial institutions,

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[3] http://www.cleanenergydurham.org/ (accessed November 20, 2012)

governments, residents) value the sustainability initiatives led by the buyer. Although the buyer cannot directly exert any sustainability effort, she can provide support to the seller in the form of money, knowledge, equipment, information, etc. We study the impact on effort and support decisions of key buyer's and seller's characteristics such as the seller's benefit and cost of effort, and the cost and efficiency of buyer's support. We also investigate how stakeholders' pressure for sustainability efforts impact effort and support decisions.

Buyer's and seller's decisions may have an impact on competing buyers and sellers. In the Apple-Foxconn example, Apple's competing buyers may face stronger stakeholder pressure to lead similar sustainability initiatives. In addition, other Foxconn buyers might free-ride on Apple's support to Foxconn. In the Clean Energy Durham example, the sustainability strategy of a particular municipality may increase the pressure on nearby communities to support similar sustainability initiatives that make them equally or more attractive to current and potential businesses and residents.

To capture the impact on other buyers and sellers, we extend our basic buyer-seller model to consider the case of two buyers who compete on sustainability performance in two possible network structures. In the first structure, the two buyers share the same seller, while in the second structure the two buyers have separate sellers. We determine conditions on stakeholder, buyer, and seller characteristics that may lead buyers and sellers to prefer one network structure over the other.

A significant amount of literature recognizes the importance to companies of addressing sustainability issues in their operations. Seuring and Müller (2008) surveyed 191 papers published between 1994 and 2007 addressing sustainability issues in supply chain management. They point out that about 74% of the papers focus on the environmental dimension of sustainability, 11% addresses the social dimension, and the remaining 15% incorporate both dimensions into the analysis. Regarding the research methodology used, they notice that more than 60% of the papers are cases or surveys, while only about 10% of the papers use mathematical models. Most of these modeling papers focus on the construction of metrics to quantify the environmental and social performance of the companies (e.g. Clift 2003, Foran et al. 2005, Noci 1997, Sarkis 2003).

Our work contributes to the small but growing literature that uses mathematical models to address sustainability decisions. Our modeling approach has similarities with that used in several papers addressing process improvement and quality decisions in the supply chain. Goupta and Loulou (1998) develop a model to determine the optimal channel structure, transfer and retail prices, and R&D levels in a four-stage game where manufacturers can reduce their production cost through process improvements. Bernstein and Kök (2009) also study cost reduction through process improvement efforts, considering the case where efforts are made by suppliers over the life cycle of a product. These two papers consider process improvements made by only one of the parties in the supply chain. Similar to our work, there are a number of papers that consider settings where both buyers and suppliers can impact improvement efforts. Harhoff (1996) examines the case of a monopolistic supplier who can contribute to improve product quality by creating knowledge spillovers that manufacturers (i.e. buyers) use as a substitute for their own R&D effort. Zhu et al. (2007) propose a model where both the buyer and the supplier can invest in quality-improvement efforts. Finally, Kim and Nettesine (2011) consider a setting where the manufacturer and the supplier engage in a collaborative effort to reduce uncertainty about component production cost. Their focus is on the impact of information asymmetry and contracting strategies on parties' incentives to collaborate.

There are some key differences between our framework and models from the process and quality improvement literature. Instead of focusing on the supply chain *per se*, our framework applies to the case of a generic buyer that outsources sustainability efforts to a generic seller. The buyer and seller could be companies, government agencies, NGOs, or any other type of organization. Additionally, while quality and process-improvement efforts usually generate benefits either through reducing the production cost or through increasing price or demand, sustainability efforts may generate multiple benefits simultaneously. For example, consider a supplier implementing energy efficiency initiatives. On the one hand, the supplier will obtain a direct benefit through electricity cost savings. On the other hand, the buyer contracting with that supplier may also obtain a benefit (e.g. increase on demand, lower taxes, access to “green” investors, etc.) for having a “greener” supplier. Kleindorfer et al. (2005) mention corporate image improvement, regulatory compliance, liability limitation, community relations improvement, employee health and safety improvement, and customer relations enhancement as additional drivers of sustainability efforts. Haanaes et al. (2011) report the results of a survey of over 3000 managers about sustainability and innovation. Nearly half of the respondents ranked improved brand reputation as the most important sustainability benefit ahead of reduced cost due to energy efficiency, increased competitive advantage, and reduced cost through materials and waste efficiencies. Other sustainability benefits ranked in the survey were access to new markets, increased market share, improved perception of management of the company, improved regulatory compliance, and improved ability to attract top talent.

Finally, a number of papers discuss conceptual or empirical evidence of the impact of stakeholders, seller, and buyer characteristics on the adoption of particular sustainability initiatives. Brun and Gereffi (2011) use Global Value Chain (GVC) analysis to study the adoption of energy efficiency practices in companies. They conclude that firms in consumer-products supply chains, operating in energy-intensive industries, are most likely to adopt energy-efficiency improvements because of the associated cost-savings and marketing value. Jira and Toffel (2012) review several papers that study how the adoption of environmental practices among suppliers is influenced by factors such as environmental regulations, suppliers’ resources and assets, number of buyers,

buyers' technical assistance and training to suppliers, and duration of the supplier-buyer relationship. The impact of several of these factors can be captured in our model.

## 3.2 Model Description

We consider a buyer that supports a seller to implement sustainability initiatives. The seller decides to exert a level of sustainability effort, $x$, for which she pays a cost, $c(x)$, and receives a direct benefit, $r(x)$. The function $r(x)$ may include a wide range of sustainability benefits that the seller may receive as a result of the effort. In the Apple-Foxconn example from the first section, the direct benefit for Foxconn may include cost reduction due to energy efficiency or waste management, operational risk reduction, and greater employee productivity. In the Clean Energy Durham example, benefits for Clean Energy Durham beyond payment for its services may include reputation enhancement and more effective fundraising. We assume $c(\cdot)$ is a linear function with marginal cost $c^0$, while $r(\cdot)$ is an increasing and concave function. Specifically, we define $r\left(x\right)=\bar{r}(1-e^{-\alpha x})$. In this function, $\bar{r}>0$ represents the maximum benefit that the seller can obtain through sustainability efforts. This parameter depends on factors such as the size of the seller and the type of activities that she does. For example, larger firms operating in energy-intensive industries may be expected to have a higher $\bar{r}$ because of opportunities for energy efficiency. Parameter $\alpha>0$ can be seen as a measure of the seller's efficiency with which she obtains direct benefits from sustainability efforts. This parameter depends on the portfolio of sustainability projects that the seller can implement. On the one hand, initiatives such as energy efficiency and recycling may produce a high direct benefit to the seller through cost reduction. On the other hand, projects such as improving education may generate a lower direct benefit to the seller. All else being equal we would expect that a seller with a portfolio dominated by the first type of sustainability project would have a higher $\alpha$ than a seller with a portfolio dominated by the second type of project. Note that a higher $\alpha$ implies a higher benefit per unit of effort, which in turn implies a lower effort per unit of benefit achieved.

Although the buyer can exert no sustainability effort, she can provide support to her seller to decrease the cost of the seller's effort. We denote the buyer's support by $u$, and assume it decreases the marginal cost of the seller's effort from $c^0$ to $c^S(u)$. Specifically, we define $c^S(u) = c^0 e^{-\beta u}$, which is a decreasing and convex function. If the buyer provides no support, the seller's cost of effort does not change from its original level, $c^0$. If the buyer provides support $u$, the seller's cost decreases according to efficiency rate $\beta > 0$. Furthermore, we assume the buyer pays a cost $c^b$ for every unit of support provided to her seller.

The buyer's stakeholders can observe the sustainability effort exerted by the buyer's seller. This assumption is not as strong as it may seem. The number of companies willing to measure, monitor, and report their sustainability performance increases every day. According to The Global Reporting Initiative (GRI), 95 percent of the world's 250 biggest companies now report their sustainability performance.[4] In the specific case of brand-owning companies, these companies typically monitor and report the sustainability performance of their suppliers. For example, Apple leads auditing programs across their entire supply chain to evaluate its supplier's performance in sustainability aspects such as labor and human rights, worker health and safety, environmental impact, and worker education and development. The results for the last six years are available at Apple's website.[5] In addition to companies' own reporting, there exist agencies such as FTSE and the Dow Jones Sustainability Indexes that evaluate and report companies' sustainability performance. The participation of these organizations generally adds transparency and credibility to the reported results. Thus, under the assumption that the buyer's stakeholders can observe sustainability efforts, we represent the "price" that they are willing to pay for every unit of effort as a linear decreasing function, $p(x) = a - bx$, where $a, b > 0$. In this function, $a$ represents the initial stakeholder's willingness to pay for sustainability efforts, while $b$ determines the rate at which this willingness to pay decreases as the effort increases. It should be noted this "price" for sustainability efforts includes a wide range of incentives provided by stakeholders to buyers such as increase in demand, the price premium paid by

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[4] Source: www.globalreporting.org (press release of March 8, 2012). GRI is a non-net benefit organization that provides companies and organizations with a comprehensive sustainability reporting framework that is widely used around the world.

[5] http://www.apple.com/supplierresponsibility/ (accessed March 27, 2012)

consumers, tax benefits, greater access to capital and better financing sources, and risk reduction.

Within this setting, we model buyer and seller decision problems as a Stackelberg game, with the buyer as the leader. The buyer moves first and offers a support $u$ to the seller. After observing the buyer's decision, the seller chooses the level of sustainability effort $x$. Our goal is to determine the equilibrium decisions and analyze how they depend on the key parameters of our model. We first present the results for the case of a monopolistic buyer, and then study the case of two buyers that compete on the sustainability performance of their sellers.

## 3.3 One Buyer – One Seller

We first consider the case of a monopolistic buyer who supports a single seller to implement sustainability initiatives.

We denote by $\pi^S\left(x|u\right)$ the seller's net benefit given buyer's support $u$. Thus, seller's decision problem is to choose the level of sustainability effort $x \geq 0$ that maximizes her net benefit function

$$\pi^S\left(x|u\right) = r\left(x\right) - c^s\left(u\right)x, \qquad (3.1)$$

where $r\left(x\right) = \overline{r}(1-e^{-\alpha x})$ and $c^S\left(u\right) = c^0 e^{-\beta u}$. Note that $\pi^S\left(\cdot|u\right)$ is concave because $\alpha$ and $\overline{r}$ are positive constants.

Let $A^0 = \frac{1}{\alpha}\ln\left(\frac{\alpha\bar{r}}{c^0}\right)$ and $\gamma = \frac{\beta}{\alpha}$. Solving the seller's maximization problem, we find that the seller's best response function is given by:

$$x(u) = \begin{cases} A^0 + \gamma u, & \text{if } u > -\frac{A^0}{\gamma}, \\ 0, & \text{otherwise.} \end{cases} \qquad (3.2)$$

Note that $\gamma$ represents the relative efficiency of buyer´s support with respect to the efficiency of seller's effort, while $A^0$ determines the effort that the seller would exert without buyer's support. If the maximum benefit she can obtain, weighted by her efficiency, is greater than her marginal cost, then $A^0$ is positive. In this case, the seller will exert an effort $A^0$ even if the buyer does not provide any support. Additionally, every unit of buyer's support will be converted into effort according to the relative efficiency rate, $\gamma$. Conversely, if $A^0$ is negative, the seller will exert a positive effort only if the effective support is high enough to compensate for the loss resulting from providing effort with no support.

Anticipating the seller's decision, the buyer chooses the level of support for the seller. We denote the buyer's net benefit by $\pi^B(u)$. Thus, the buyer chooses the level of support $u \geq 0$ that maximizes her net benefit function

$$\pi^B(u) = p(x)x - c^b u, \qquad (3.3)$$

where $p(x) = a - bx$ and $x$ is given by (3.2).

The following proposition presents the optimal decisions and the corresponding optimal net benefits. All proofs can be found in the Appendix.

PROPOSITION 3.1. *The optimal effort and support decisions,* $x^*$ *and* $u^*$*, vary for four different regions characterized by* $A^0$*,* $a$*,* $c^b$*, and* $\gamma$*.*

*Let* $\Omega_1 = \left\{ \left( A^0, a, c^b, \gamma \right) : A^0 > 0,\, a - c^b / \gamma > 2bA^0 \right\}$ *and*

$\Omega_2 = \left\{ \left( A^0, a, c^b, \gamma \right) : A^0 \le 0,\, a - c^b / \gamma > \left( -4bc^b A^0 / \gamma \right)^{1/2} \right\}$*. If* $\left( A^0, a, c^b, \gamma \right) \in \left\{ \Omega_1 \cup \Omega_2 \right\}$*, then*

$$x^* = \frac{a - c^b / \gamma}{2b},\ u^* = \frac{a - c^b / \gamma - 2bA^0}{2b\gamma}. \qquad (3.4)$$

*The corresponding optimal net benefits are given by*

$$\pi^{S*} = \overline{r}\left[ 1 - \left( 1 + \frac{\alpha}{2b}\left( a - c^b / \gamma \right) \right) e^{-\frac{\alpha}{2b}\left( a - c^b / \gamma \right)} \right],\quad \pi^{B*} = \frac{\left( a - c^b / \gamma \right)^2}{4b} + \frac{c^b A^0}{\gamma}. \qquad (3.5)$$

*Let* $\Omega_3 = \left\{ \left( A^0, a, c^b, \gamma \right) : A^0 > 0,\ a - c^b / \gamma \le 2bA^0 \right\}$*. If* $\left( A^0, a, c^b, \gamma \right) \in \Omega_3$*, then* $x^* = A^0$ *and* $u^* = 0$*. The corresponding optimal net benefits are given by*

$$\pi^{S*} = \overline{r} - c^0 / \alpha - c^0 A^0,\ \pi^{B*} = A^0 \left( a - bA^0 \right) \qquad (3.6)$$

*Let* $\Omega_4 = \left\{ \left( A^0, a, c^b, \gamma \right) : A^0 \le 0,\, a - c^b / \gamma \le \left( -4bc^b A^0 / \gamma \right)^{1/2} \right\}$*. If* $\left( A^0, a, c^b, \gamma \right) \in \Omega_4$*, then* $x^* = 0$ *and* $u^* = 0$*. The corresponding optimal net benefits are* $\pi^{S*} = 0$ *and* $\pi^{B*} = 0$*.*

Figure 11 shows the four regions described in Proposition 3.1. In regions $\Omega_1$ and $\Omega_3$, the seller has an incentive to exert an effort $A^0$, even without any support from the buyer. Moreover, in region $\Omega_1$, stakeholders' willingness to pay is high enough so the buyer will provide the support required to increase seller's effort up to the level $x^*$ defined in (3.4). Conversely, in regions $\Omega_2$ and $\Omega_4$, the seller does not have any internal incentive to exert an effort. If stakeholders' willingness to pay is high enough, as in region $\Omega_2$, then the buyer will provide the support required to induce the effort $x^*$ defined in (3.4). However, if the buyer does not provide any support, as in region $\Omega_4$, then the seller does not exert any effort. Finally, note that the buyer's support required to induce an effort $x^*$ is higher in $\Omega_2$ than in $\Omega_1$.

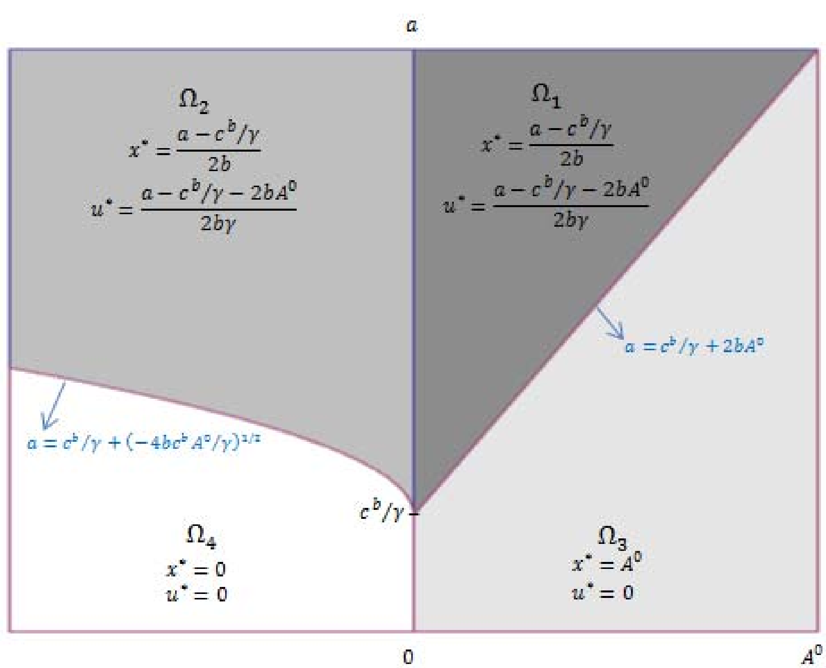


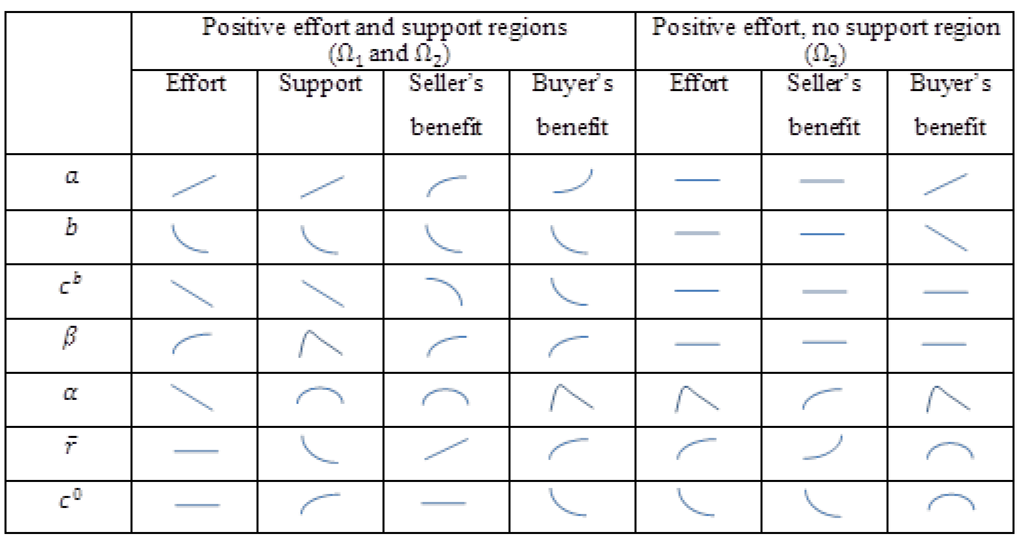

**Figure 11: Structure of the Optimal Effort and Support Decisions.**

Looking at the impact of possible changes in the parameters on the optimal decisions and optimal net benefits, we first note that the buyer's support and the seller's effort, as well as the buyer and seller net benefits, generally increase if the stakeholders' price for efforts increase (higher $a$, lower b) or if the cost of the buyer's support decrease. The efficiency of the buyer's support, $\beta$, only plays a role if the buyer's support is positive, i.e., regions $\Omega_1$ and $\Omega_2$. As $\beta$ increases, the buyer's support is more efficient in reducing seller's cost of effort. Therefore, the seller's optimal effort increases. The buyer's optimal support is increasing in $\beta$ for $\beta < 2\alpha c^b / (a - 2bA^0)$. After that point, the buyer's support needed to induce the optimal seller effort decreases as she becomes more efficient. Consequently, both the seller's and the buyer's net benefits are increasing in $\beta$. Furthermore, the net benefits increase at a decreasing rate, because both the seller's direct benefit and the stakeholder's price have upper bounds.

Finally, we study the impact of the three parameters that characterize the seller, $\overline{r}$, $c^0$, and $\alpha$. In region $\Omega_3$, where the buyer provides no support, increasing $\overline{r}$ or decreasing $c^0$ increase both the seller's optimal effort and optimal net benefit. In regions $\Omega_1$ and $\Omega_2$, $\overline{r}$ and $c^0$ do not affect optimal effort because, in these two regions, the buyer will provide the support needed to achieve the optimal effort, regardless of the seller's characteristics. Therefore, decreasing $\overline{r}$ or increasing $c^0$ (within $\Omega_1$ and $\Omega_2$) will increase the buyer's support and decrease the buyer's net benefit.

To analyze the impact of changes in the seller's efficiency, $\alpha$, let us start with region $\Omega_3$, where the seller's optimal effort is equal to $A^0$. In this case, as $\alpha$ increases, the seller initially increases her optimal effort and then decreases it when $\alpha > c^0 e / \overline{r}$. In regions $\Omega_1$ and $\Omega_2$, not only the seller's optimal effort but also the buyer's optimal support is affected by changes in $\alpha$. As $\alpha$ increases, the seller is more efficient in obtaining a direct benefit from her sustainability effort. Therefore, the support required from the buyer to increase her effort level is greater. In other words, as $\alpha$ increases, support is transformed into effort at a lower rate. For low values of $\alpha$, the buyer tries to compensate for the increase in $\alpha$ by increasing her support. After some point (specifically, $\alpha = \left[ a\beta + \left( a^2\beta^2 - 16(c^b)^2 b\beta \right)^{1/2} \right] / 4c^b$), increasing support costs the buyer too much, and she starts to decrease her support. Consequently, the buyer's optimal net benefit is increasing in $\alpha$ for $\alpha \leq \left[ a\beta - \left( a^2\beta^2 - 8c^b b\beta \right)^{1/2} \right] / 2c^b$, and decreasing in $\alpha$ when $\alpha$ is greater than this critical value. Regarding the impact on the seller's decision, we can see that the seller's optimal effort decreases as $\alpha$ increases, as a consequence of the decrease on the relative effectiveness of the buyer's support. The seller's optimal net benefit initially increases as $\alpha$ increases, achieves a maximum at $\alpha = a\beta / 2c^b$, and then starts to decrease. This is a consequence of the decrease in the optimal buyer's support after some $\alpha$. This result implies that being "too efficient" in obtaining benefit from sustainability efforts may hurt the seller. Although this may seem a bit counterintuitive, recall that being more efficient in this context means that the seller has a portfolio of sustainability initiatives that lead to the maximum direct benefit with less sustainability effort. Because the buyer's stakeholders' care about sustainability effort, not the seller's direct benefit, this decrease in the sustainability effort hurts the buyer's net benefit. Consequently, the buyer provides less support, which in turn may also hurt the seller's net benefit. Table 1 summarizes the general shape of the impacts of seller's and buyer's characteristics on the optimal decisions and optimal net benefits. For example, the table shows that, in regions $\Omega_1$ and $\Omega_2$, when $a$ increases, the optimal seller's effort $x^*$ increases at a lineal rate while the optimal seller's net benefit $\pi^{S*}$ also increases but it does it at a decreasing rate.

**Table 1: Optimal Effort, Support, and Net Benefits under One-at-a-time Changes on Key Parameters.**



## 3.4 Competing Buyers

In this section we study the case of two buyers who compete for stakeholders' payments based on the sustainability effort exerted by their sellers. Each buyer needs only one seller to implement sustainability initiatives. Thus, we consider two cases. In the first case both buyers outsource from the same seller, whereas in the second case the buyers' outsource two different sellers.

Buyer competition adds some new elements into our problem. Particularly, we focus our attention on two types of spillovers that may have an impact on the optimal effort and support decisions. The first type of spillover, which we called *support spillover*, is generated when buyers shared a seller. Given the nature of the seller's sustainability efforts, it may be difficult to associate them with a particular buyer. For example, if a seller improves labor conditions, it is reasonable to assume that the improvement would affect all workers, not just those working in the initiatives associated to a specific buyer. Therefore, a buyer's support to induce a higher effort by her seller may also benefit other seller's buyers.

The second type of spillover, which we called *effort spillover*, is generated when there are multiple sellers. In this case, the sustainability effort exerted by one seller can affect other sellers. For example, if a seller improves labor conditions for her workers, the pressure on other sellers to do the same may increase, which in turn may represent an additional cost. In another example, consider a seller that creates a program to improve health and education in the community where she operates. Other sellers operating in the same community may benefit from operating in a better social environment and having access to more qualified labor.

We now describe the model for the cases of shared-seller and separate-seller networks, and we compare the optimal support and effort decisions in these two cases. To simplify our analysis, we focus on the case of identical buyers and identical sellers.

### 3.4.1 Shared-Seller Network

In this case, we consider two identical buyers that outsource sustainability efforts from the same seller. We use the same functional forms as in the previous section. Thus, the seller chooses the level of sustainability effort $x \geq 0$ that maximizes her net benefit

$$\pi^S\left(x \middle| u_1, u_2\right) = \hat{r}(1-e^{-\alpha x}) - c^0 e^{-\beta(u_1+u_2)} x, \qquad (3.7)$$

where $u_1$ and $u_2$ represent the support received from buyers 1 and 2, respectively. Note that the support efficiency rate, $\beta$, is the same for the two buyers. Also we denote the maximum direct benefit that the seller can achieve in this case by $\hat{r}$, which can be different from $\overline{r}$, defined in the case of a monopolistic buyer. Solving the seller's maximization problem in this case, we find that the seller's best response function is given by:

$$x(u_1, u_2) = \begin{cases} \hat{A}^0 + \gamma(u_1 + u_2), & if\ u_1 + u_2 > -\dfrac{\hat{A}^0}{\gamma}, \\ 0, & otherwise, \end{cases} \qquad (3.8)$$

with $\hat{A}^0 = \frac{1}{\alpha} \ln\left(\frac{\alpha \hat{r}}{c^0}\right)$. Note that the seller's best response function has a similar form to that in the previous section, but now depends on the sum of the support received from the two buyers.

Anticipating the seller's decision, the buyers simultaneously choose their levels of support $u_i$, $i = 1,2$, that maximize her net benefit

$$\pi_i^B\left(u_i \middle| u_{-i}\right) = \left(a - bx\right)\frac{x}{2} - c^b u_i, \qquad (3.9)$$

Note that, according to (3.9), each buyer receives a payment for half of the seller's sustainability effort. The underlying assumption in this case is that, in addition to observing the seller's sustainability effort, stakeholders also know that the buyers share the same seller. Therefore, each buyer "claims" half of the seller's effort.

To simplify our analysis and focus our attention on the comparison between the shared-seller and different-sellers networks, we restrict our analysis to the case where $\hat{A}^0 > 0$. This implies that the seller exerts a positive effort even without any support from the buyers. Solving the buyers' maximization problem, we find that buyer $i$'s best response function is given by

$$u_i\left(u_{-i}\right) = \begin{cases} K - u_{-i}, & if\ K - u_{-i} > 0, \\ 0, & otherwise, \end{cases} \qquad (3.10)$$

where $K = \frac{1}{2b\gamma}\left(a - 2c^b / \gamma - 2b\hat{A}^0\right)$.

Proposition 3.2 presents the optimal decisions and the corresponding optimal net benefits for the shared-seller network.

PROPOSITION 3.2. *Assume that* $\hat{A}^0 > 0$ *and* $K > 0$. *The unique symmetric Nash equilibrium of the shared-seller network game is given by*

$$x^* = \frac{a - 2c^b / \gamma}{2b}, \; u_1^* = u_2^* = \frac{a - 2c^b / \gamma - 2b\hat{A}^0}{4b\gamma}. \qquad (3.11)$$

*The corresponding optimal net benefits are given by*

$$\pi^{S*} = \hat{r}\left[1 - \left(1 + \frac{\alpha}{2b}\left(a - 2c^b / \gamma\right)\right)e^{-\frac{\alpha}{2b}\left(a - 2c^b / \gamma\right)}\right],$$

$$\pi_1^{B*} = \pi_2^{B*} = \frac{a\left(a - 2c^b / \gamma\right)}{8b} + \frac{c^b \hat{A}^0}{2\gamma}. \qquad (3.12)$$

As we observe in (3.8), the seller decides about the optimal effort based on the sum of the buyers' supports. This implies that the buyers' supports are substitutes for each other, which leads to multiple equilibria in the game. Specifically, the equilibria set consist of $u_1, u_2 \geq 0$ such that $u_1 + u_2 = K$. Proposition 3.2 presents the symmetric equilibrium where $u_1 = u_2 = K / 2$.

To be able to compare the results of this section with those obtained in the case of a monopolistic buyer, we need to make some assumptions about the value of the parameters in each case. Let us initially assume that the value of the parameters do not change from the monopolistic buyer case to the shared-seller case; in particular $\hat{r} = \overline{r}$. In the shared-seller case, the buyers receive a payment for only half of the effort exerted by the seller. This leads to a decrease in the equilibrium effort relative to the monopolistic buyer case, as we can see by comparing (3.4) and (3.11). Particularly, note that the optimal effort in (3.11) is equivalent to the solution in (3.4) but with an "equivalent cost" equal to twice the cost of support, $c^b$. As a consequence, the sum of the supports that the seller receives from the two buyers is also lower. This reduction in the level of effort and supports leads to a decrease in the optimal net benefit for both the buyers and the seller relative to the monopoly case, as we can see by comparing (3.5) and (3.12).

We could alternatively assume that some parameters change in the shared-seller network. For example, it seems reasonable to assume that the total cost savings resulting from an energy efficiency program are proportional to the size of the operations of the seller. If we additionally assume that the size of the operations is proportional to the number of buyers served, then the maximum direct benefit that the seller can achieve is higher when she is working with two buyers, i.e., $\hat{r} > \overline{r}$. In this case, both the seller and the buyer may be better off in the shared-seller network relative to the monopolistic buyer case, as we can see by comparing equations (3.5) and (3.12).

### 3.4.2 Separate-Sellers Network

We now study a setting with two identical buyers that outsource sustainability efforts from two separate but identical sellers. Thus, buyer 1 outsources efforts from seller 1, and buyer 2 outsources from seller 2. As we previously mentioned, in this case the effort exerted by one of the sellers may have an impact on the benefit of the other seller. We model this by introducing a parameter $\delta \in (-1,1)$ to reflect the fraction of the effort of a given seller that spills over to the other seller. A positive $\delta$ means that one seller's effort benefits the other seller, while a negative $\delta$ implies that it hurts the other seller. Therefore, in the second stage of the game the sellers simultaneously choose their effort levels given the buyers' support. An important assumption we make in this case is that the buyers' supports are observable by both sellers. Thus, seller $i$'s net benefit function in this case is given by

$$\pi_i^S\left(x_i \middle| x_{-i}, u_i\right) = \tilde{r}\left(1 - e^{-\alpha(x_i + \delta x_{-i})}\right) - c^0 e^{-\beta u_i} x_i. \quad (3.13)$$

We denote the maximum direct benefit that each seller can achieve in this case by $\tilde{r}$, which can be different from $\overline{r}$ and $\hat{r}$ defined in the previous sections. Also note that the effort efficiency rate, $\alpha$, is the same for both sellers, given our assumption that they are identical. Solving seller $i$'s maximization problem in this case, we find that seller $i$'s best response function is given by:

$$x_i\left(x_{-i}, u_i\right) = \begin{cases} \tilde{A}^0 + \gamma u_i - \delta x_{-i}, & if\ u_i > \dfrac{-\tilde{A}^0 + \delta x_{-i}}{\gamma}, \\ 0, & otherwise, \end{cases} \quad (3.14)$$

where $\tilde{A}^0 = \dfrac{1}{\alpha}\ln\left(\dfrac{\alpha\tilde{r}}{c^0}\right)$. Thus, the equilibrium of the effort game played by the sellers after knowing the buyers' support is given by

$$x_i\left(u_1, u_2\right) = \begin{cases} \dfrac{\tilde{A}^0}{(1+\delta)} + \dfrac{\gamma(u_i - \delta u_{-i})}{(1-\delta^2)}, & if\ u_i - \delta u_{-i} > -\dfrac{\tilde{A}^0(1-\delta)}{\gamma}, \\ 0, & otherwise. \end{cases} \quad (3.15)$$

As in the shared-seller network we focus on the case where sellers have an incentive to exert a positive effort even without buyers' support, i.e., $\tilde{A}^0 > 0$. Anticipating the sellers' decisions, the buyers simultaneously choose their levels of support $u_i$, $i = 1,2$, that maximize their net benefits

$$\pi_i^B\left(u_i \middle| u_{-i}\right) = \left(a - b(x_1 + x_2)\right) x_i - c^b u_i, \qquad (3.16)$$

Solving the buyers' maximization problem, we find that buyer $i$'s best response function is given by

$$u_i\left(u_{-i}\right) = \begin{cases} \tilde{K} - \dfrac{(1-\delta)}{2} u_{-i}, & if\ 2\tilde{K} - (1-\delta)u_{-i} > 0, \\ 0, & otherwise, \end{cases} \qquad (3.17)$$

where $\tilde{K} = \dfrac{1}{2b\gamma}\left((1+\delta)a - (1-\delta)(1+\delta)^2 c^b / \gamma - (3-\delta)b\tilde{A}^0\right)$.

Proposition 3.3 presents the optimal decisions and the corresponding optimal net benefits for the separate-sellers network.

PROPOSITION 3.3. *Assume that* $\tilde{A}^0 > 0$ *and* $\tilde{K} > 0$. *The Nash equilibrium of the separate-sellers network game is given by*

$$x_1^* = x_2^* = \frac{a - (1-\delta^2)c^b / \gamma}{(3-\delta)b},$$

$$u_1^* = u_2^* = \frac{(1+\delta)a - (1-\delta)(1+\delta)^2 c^b / \gamma - (3-\delta)b\tilde{A}^0}{(3-\delta)b\gamma}. \qquad (3.18)$$

*The corresponding optimal net benefits are given by*

$$\pi_1^{S*} = \pi_2^{S*} = \tilde{r}\left[1 - \left(1 + \frac{\alpha}{(3-\delta)b}\left(a - (1-\delta^2)c^b / \gamma\right)\right)e^{-\frac{\alpha(1+\delta)}{(3-\delta)b}\left(a-(1-\delta^2)c^b/\gamma\right)}\right],$$

$$\pi_1^{B*} = \pi_2^{B*} = \frac{(1+\delta)\left[a - (1-\delta^2)c^b / \gamma\right]^2}{(1-\delta)(3-\delta)^2 b} - \frac{\delta a[a - \left(1-\delta^2\right)c^b / \gamma]}{(1-\delta)(3-\delta)b} + \frac{c^b \tilde{A}^0}{\gamma}. \qquad (3.19)$$

The results for the separate-sellers network have a structure similar to those that would arise in a typical Cournot competition. If we assume that all parameters of the model have the same value and, additionally, that there is no effort spillover, then the optimal sustainability effort of each seller is lower than that in the original "single buyer - single seller" case. However, the sum of the sellers' efforts is higher than the optimal effort in the original case, as we can see by comparing equations (3.4) and (3.18). Similarly, the buyer's support is also lower. This translates into lower optimal net benefit for both buyers and sellers.

Finally, we analyze the impact of the effort spillover parameter on the optimal decisions. First, note that a positive $\delta$, which represents a positive externality between sellers' efforts, decreases the level of effort that the seller exerts without buyer's support from $\tilde{A}^0$ to $\tilde{A}^0 / (1+\delta)$. Thus, each buyer needs to increase her optimal support in order to increase the optimal effort. Therefore, each seller's net benefit is increasing in $\delta$, while the buyer's net benefit is decreasing in $\delta$. A negative $\delta$ has the opposite effects.

Table 2 summarizes the direction of the changes in the effort, the support, and the net benefits in the shared-seller and separate-sellers networks relative to the one buyer-one seller case. Note that while buyers and sellers would generally prefer the monopolistic case, stakeholders would prefer the separate-sellers case because the total sustainability effort is higher than in the other type of networks.

**Table 2: Competing Buyers vs. Monopolistic Buyer.**

| | Shared-seller | Separate-sellers (no effort spillover) |
|---|---|---|
| Total support received by each seller | ↓ | ↓ |
| Seller's effort | ↓ | ↓ |
| Total effort in the network | ↓ | ↑ |
| Seller's net benefit | ↓ | ↓ |
| Buyer's net benefit | ↓ | ↓ |

Direction of the changes in the effort, the support, and the net benefits in the shared-seller and separate-sellers networks relative to the one buyer-one seller case.

### 3.4.3 Shared-Seller vs. Separate-Sellers

We now compare the results obtained in the previous two sections for the shared-seller and separate-seller networks. We focus our analysis on the performance variables relevant to the agents in the problem. On the one hand, buyers and sellers want to maximize their net benefit. On the other hand, the buyers' stakeholders care about the total sustainability effort exerted by all the sellers in the network. Proposition 3.4 presents the conditions under which these measures are higher in one network than in the other. We initially assume that all the corresponding parameters are equal in the two cases, and that there is no effort spillover. Then, we discuss how the comparison is affected when the upper limit on the direct benefit of the sellers is different in the two cases, and when there is an effort spillover in the separate-sellers case.

PROPOSITION 3.4. *Let* $\hat{A}^0, \tilde{A}^0, K, \tilde{K} > 0$. *Additionally, let* $\hat{A}^0 = \tilde{A}^0$ *and* $\delta = 0$. *Then:*

1. *The combined sustainability effort exerted by sellers is always lower in the shared-seller network than in the separate-sellers network.*
2. *The seller´s optimal net benefit is higher in the shared-seller network than in the separate-sellers network if* $a \geq \dfrac{4c^b}{\gamma}$.
3. *The buyer´s optimal net benefit is higher in the shared-seller network than in the separate-sellers network if* $a \geq \dfrac{4c^b}{\gamma} + 36b\ln\left(\dfrac{\alpha\tilde{r}}{c^0}\right)$.

According to the first result of Proposition 3.4, the combined effort exerted by the two sellers in the separate-sellers case is always higher than the effort exerted by the single seller in the shared-seller case. To understand why this is the case, note that, on the one hand, the equilibrium effort in the shared-seller case has a structure similar to the monopolistic buyer case, but with each buyer receiving a payment for only half of the seller's effort. On the other hand, the buyers in the separate-sellers case play a Cournot effort game. Thus, as expected, competition increases the total effort expended in the network. Therefore, stakeholders always prefer the separate-sellers network over the shared-seller network.

The second result of Proposition 3.4 presents a condition under which the seller in the shared-seller case is better off compared to each of the sellers in the separate-sellers case. To understand this result, first note that because we are assuming that $\hat{A}^0 = \tilde{A}^0$, the difference between the seller's effort in the two cases depends on the total support that she receives from the buyer(s) in each case. From (3.11) and (3.18), we observe that the total support received by the seller in the shared-seller network is $\dfrac{a - 2c^b / \gamma - 2b\hat{A}^0}{2b\gamma}$, while the support received by each seller in the separated-sellers case is $\dfrac{a - c^b / \gamma - 3b\tilde{A}^0}{3b\gamma}$. The first expression is greater than the second one when $a \geq 4c^b / \gamma$. Therefore, increasing stakeholders' willingness to pay, increasing buyers' relative efficiency, or decreasing buyers' support cost all make the shared-seller network more attractive to the seller relative to the separate-sellers network.

The third result of Proposition 3.4 shows that in addition to $a$, $\gamma$, and $c^b$, the buyer's preference about the network structure also depends on $\tilde{r}$ and $c^0$. Lower values of $\tilde{r}$ and higher values of $c^0$ makes the shared-seller network more attractive to the buyer relative to the separate-sellers network. Furthermore, note that the value of $a$ required by the buyer to choose the shared- seller network is always higher than that required by the seller, because $36b\ln\left(\frac{\alpha\tilde{r}}{c^0}\right) > 0$. This implies that if the buyers prefer the shared-seller network, then the seller does as well. Proposition 3.4 is illustrated in Figure 12. We can see that the shared-seller network is more attractive to both buyers and sellers when stakeholders are willing to pay more for sustainability efforts (i.e. $\uparrow a, \downarrow b$), buyers are more efficient in helping their sellers (i.e. $\uparrow \beta, \downarrow c^b$), and sellers obtain a lower direct benefit from efforts ($\downarrow \alpha, \downarrow \tilde{r}, \uparrow c^0$).

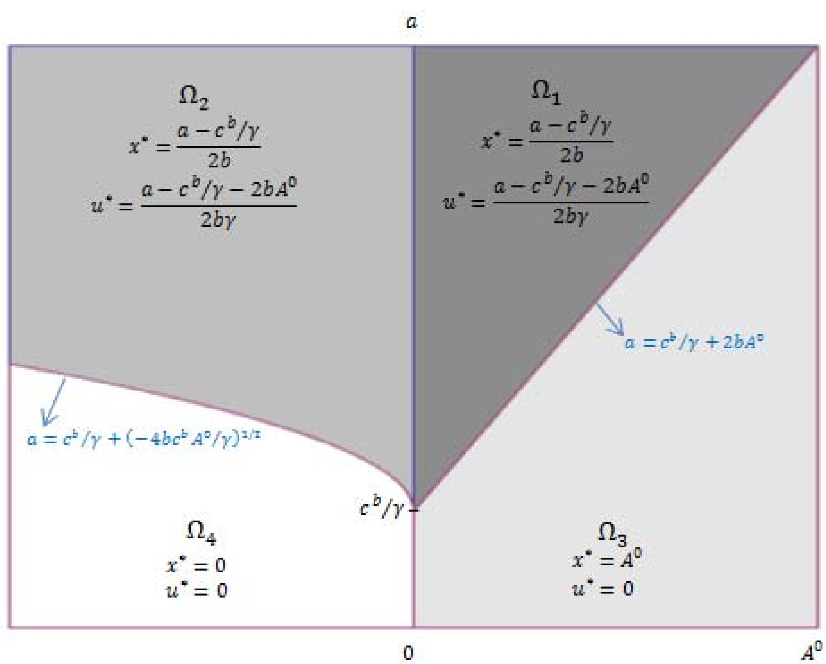



**Figure 12: Buyers and Sellers Network Preferences.**

Finally, we study how the results in Proposition 3.4 change if there is an effort spillover or if we allow $\hat{A}^0 \neq \tilde{A}^0$. First, we note that even though a negative $\delta$ decreases the combined effort exerted by the two sellers in the separate-seller network, that combined effort is still greater than the effort exerted by the single seller in the shared-seller network, for any value of $\delta$. Regarding the second result of Proposition 3.4, we noted in the previous section that the seller's net benefit in the separate-seller network is increasing in $\delta$. Thus, the seller requires higher values of $\delta$, $a$, and $\gamma$, and a lower value of $c^b$ to prefer the shared-seller network. Conversely, the buyer's net benefit is decreasing in $\delta$. Therefore, higher values of $\delta$, $c^b$, and $\tilde{r}$; and lower values of $a$, $\gamma$, and $c^0$, may lead the buyer to prefer the shared-seller network.

We now assume that $\hat{r} \neq \tilde{r}$, which implies that $\hat{A}^0 \neq \tilde{A}^0$. Particularly, we assume that $\hat{r} \geq \tilde{r}$, given that the seller is working with two buyers in the shared-seller network and only with one of them in the separate-sellers network. Assuming $\hat{r} \geq \tilde{r}$ implies that both seller and buyer net benefits in the shared-seller network are greater than the net benefits in the separate-seller network. Therefore, the higher the value of $\hat{r}$ relative to $\tilde{r}$, the lower the values of $a$, and $\gamma$, and the higher the value of $c^b$, that seller and buyer require to prefer the shared-seller network over the separate-sellers network.

## 3.5 Summary and Conclusions

Companies and other type of organizations face growing pressure to improve their sustainability performance. In this chapter, we developed a model to study the situation in which a buyer provides support to her seller to increase the seller's sustainability effort. We found that sustainability efforts generally increase when stakeholders heighten their interest in sustainability performance, when buyers are more efficient to provide support to their sellers, and when sellers manage a balanced portfolio of social and environmental projects, and not only projects with a high direct economic return. We then analyzed the situation of two buyers that compete on sustainability performance, and compare the case where buyers share a seller against the case where buyers have separate sellers. We conclude that, while stakeholders generally prefer the separate-sellers network, buyers have more incentives to share a seller when stakeholders are willing to pay more for sustainability efforts, when buyers are more efficient in helping their sellers, and when sellers obtain a lower direct benefit from sustainability efforts.

Some of our findings are consistent with empirical evidence presented in some other papers investigating the impact of stakeholders, buyers and seller's characteristics on the adoption of sustainability initiatives. Jira and Toffel (2012) cite several papers that investigate the impact of some supply chain characteristics on the adoption of particular environmental practices. As noted by Jira and Toffel (2012), Delmas and Montiel (2009) and Locke et al. (2007) found that adoption of environmental practices is more likely in countries with stronger regulatory requirements to publicly disclose pollution data. This can be captured in our model as a greater stakeholders' interest (government in this case) in sustainability performance. Similarly, they mention that Lee (2008) and Locke et al. (2009) found that suppliers are more willing to adopt sustainability practices when buyers provide technical assistance and training, engage in joint problem solving, and share best practices with their sellers. This can be represented in our model as a greater efficiency in the support from buyers to sellers. Brun and Gereffi (2011) use GVC analysis to study the adoption of energy efficiency practices in companies. They found that firms in consumer-products supply chains, operating in energy-intensive industries, and closer to the consumers, are most likely to adopt energy-efficiency improvements because of the associated cost-savings and marketing value. This can be incorporated into our model as a higher stakeholders' willingness to pay for sustainability efforts, and a higher seller's direct benefits from these efforts.

We think that combining our model with the main elements of GVC analysis could help to better understand the drivers of sustainability efforts along the supply chain. A value chain represents the set of activities in which firms engage to produce goods and services. One of the most important characteristics of a GVC is the kind of governance used by leading firms in the chain to coordinate the relationship with other firms (Gereffi et al. 2005 and Bair 2009). Gereffi et al. (2005) identify five types of value chain based on three factors: the complexity of the information and knowledge transfer required in the transactions; the ability to codify and efficiently transmit this knowledge along the chain; and the capabilities of existing suppliers to meet the buyers' requirements. We think that these three factors could be associated with some of the parameters in our model to determine which types of governance induce a better sustainability performance in the supply chain.

Finally, there are some possible extensions to our model that could be helpful to obtain additional insights in our study of sustainability efforts. First, we could study the impact of collaboration between buyers to provide support to common sellers. According to Plambeck et. al. (2012) companies such as Nike, Levi Strauss, and Adidas are working together in China to drive sustainability efforts in their common sellers. Adding this collaboration to our model would create an interesting collaboration-competition relationship between buyers. Second, it may be interesting to examine the role of NGOs to drive sustainability efforts in the supply chain. In addition to put pressure on brand buyers to improve the sustainability performance of their supply chains, NGOs may also play a key role to actually increase sustainability efforts along the chain. For example, they can facilitate monitoring and work with buyers and sellers to identify opportunities to improve their sustainability performance. An example of this, discussed by Plambeck et. al. (2012), is the collaborative relationship between Timberland, Walmart, Nike, and other buyers with the Institute of Public and Environmental Affairs in China to encourage sellers to identify and fix air and water violations. Finally, it may be worthwhile to study the case where stakeholders, buyers, and sellers have incomplete information about the network structure. This could have an impact on the payment schemes from stakeholders to buyers, as well as from buyers to sellers.

# Appendix

## *A. Proof of Results in Chapter 2*

*Proof of Proposition 2.1*

1. First, note that the reward function $r(x,y,h)$ is non-decreasing and convex in $y$ for each action $h$. Additionally, the timber price transitions $\tilde{y}_{k+1}(y_k)$ are increasing and convex in the sense of second-order, monotonic stochastic dominance (see Smith and McCardle 2002). Thus, by Proposition 5 in Smith and McCardle (2002), $J^*(x,y)$ is non-decreasing and convex in $y$.
2. Note that the reward function $r(x,y,h)$ is non-decreasing in $x$ for each action $h$. Also, volume transitions $x_{k+1}(x_k,h)$ are non-decreasing in $x$ for each action $h$. Thus, by Proposition 5 in Smith and McCardle (2002), $J^*(x,y)$ is non-decreasing in $x$.
3. Note that for a given $x$, $r(x,y,1)-r(x,y,0)=e^y x-R$ is increasing in $y$. Therefore, $r(x,y,h)$ is supermodular in $(y,h)$. Additionally, timber price transitions $\tilde{y}_{k+1}(y_k)$ are supermodular in $(y,h)$ because they do not depend on $h$. Therefore, by Proposition 5 in Smith and McCardle (2002), $J(x,y,h)$ is supermodular in $(y,h)$.

*Proof of Proposition 2.2*

1. Note that $J^*(x,y)$ is obtained after choosing the action $h$ that maximizes $J(x,y,h)$. From Proposition 1, part C, we know that, for a given $x$, $J(x,y,h)$ is supermodular in $(y,h)$. Thus, we can use Topkis's Monotonicity Theorem (Topkis 1978) to conclude that $h^*(y)$ is non-decreasing, which means that if it is optimal to harvest at a price $y$, it is also optimal to harvest at any price $y'$ such that $y' \geq y$.

2. First, note that while the reward function $r(x,y,h)$ is supermodular in $(x,h)$ for a given $y$, the transitions $x_{k+1}(x_k,h)$ are submodular in $(x,h)$. Therefore, we cannot apply Proposition 5 in Smith and McCardle 2002. Instead, we will proof by induction that, for a given $y$, $\nabla(x,y) = J(x,y,1) - J(x,y,0)$ is increasing in $x$ when $f'(x)E\left[e^{\tilde{z}}\right] \leq 1/\delta$. According to the value iteration method, $J_{k+1}(x,y) = \max\left\{\delta E\left[J_k\left(f(x),\tilde{g}(y)\right)\right], e^y x - R + \delta E\left[J_k\left(x_0,\tilde{g}(y)\right)\right]\right\}$. Let $J_0(x,y) = 0$. Thus, $J_1(x,y) = \max\left\{0, e^y x - R\right\}$ and $\nabla_1(x,y) = e^y x - R - 0$, which is increasing in $x$, for a given $y$. Assume that $\nabla_{k+1}(x,y) = e^y x - R + \delta E\left[J_k\left(x_0,\tilde{g}(y)\right)\right] - \delta E\left[J_k\left(f(x),\tilde{g}(y)\right)\right]$ is increasing in $x$, for any non-decreasing and concave function $f(.)$, and transitions of the form $\tilde{g}(y) = y + \tilde{z}$. We must show that $\nabla_{k+2}(x,y) = e^y x - R + \delta E\left[J_{k+1}\left(x_0,\tilde{g}(y)\right)\right] - \delta E\left[J_{k+1}\left(f(x),\tilde{g}(y)\right)\right]$ is increasing in

$x$ or, equivalently, that $\nabla'_{k+2}(x,y) = e^y x - \delta E\left[J_{k+1}\left(f(x), \tilde{g}(y)\right)\right]$ is increasing in $x$. Given that the expectation is an increasing function, we can interchange the order of the maximization and the expectation and show that

$\nabla''_{k+2}(x,y) = e^y x - \delta J'_{k+1}\left(f(x), y\right)$ is increasing in $x$, with

$J'_{k+1}\left(f(x), y\right) = \max\left\{\delta E\left[J_k\left(f^2(x), \tilde{g}^2(y)\right)\right], E\left[e^{\tilde{g}(y)} f(x)\right] - R + \delta E\left[J_k\left(x_0, \tilde{g}^2(y)\right)\right]\right\}$

If $J'_{k+1}\left(f(x), y\right) = \delta E\left[J_k\left(f^2(x), \tilde{g}^2(y)\right)\right]$,

$\nabla''_{k+2}(x,y) = e^y x - \delta^2 E\left[J_k\left(f^2(x)), \tilde{g}^2(y))\right)\right]$. Let add and subtract $E\left[e^{\tilde{g}(y)} f(x)\right]$.

By the induction hypothesis, $E\left[e^{\tilde{g}(y)} f(x)\right] - \delta^2 E\left[J_k\left(f^2(x)), \tilde{g}^2(y))\right)\right]$ is increasing in $x$. Thus, $\nabla''_{k+2}(x,y)$ will be increasing in $x$ if $e^y x - \delta E\left[e^{\tilde{g}(y)} f(x)\right]$ is increasing in $x$. Note that $\delta E\left[e^{\tilde{g}(y)} f(x)\right] = \delta f(x) e^y E\left[e^{\tilde{z}}\right]$. Therefore $\nabla''_{k+2}(x,y)$ is increasing in $x$ if $f'(x) \leq 1/\delta E\left[e^{\tilde{z}}\right]$ (assuming that $f(.)$ is differentiable). If

$J'_{k+1}\left(f(x), y\right) = E\left[e^{\tilde{g}(y)} f(x)\right] - R + \delta E\left[J_k\left(x_0, g^2(y)\right)\right]$, then

$\nabla''_{k+2}(x,y) = e^y x - \delta\left(E\left[e^{\tilde{g}(y)} f(x)\right] - R + \delta E\left[J_k\left(x_0, \tilde{g}^2(y)\right)\right]\right)$, which is increasing in $x$ if $f'(x) \leq 1/\delta E\left[e^{\tilde{z}}\right]$, as shown in the previous paragraph.

*Proof of Proposition 2.3*

The results in this proposition can be proved in a similar way to those of Proposition 2.1.

*Proof of Proposition 2.4*

The results in this proposition can be proved in a similar way to those of Proposition 2.2.

## B. Proof of Results in Chapter 3

*Proof of Proposition 3.1*

Anticipating the seller's response function given by (3.2), the buyer chooses the support level $u \geq 0$ that maximizes her benefit function

$$\pi^B(u) = \begin{cases} A^0\left(a - bA^0\right) + \left(a - 2bA^0 - c^b / \gamma\right)\gamma u - b\gamma^2 u^2 & \text{if } u \geq -A^0 / \gamma \\ -c^b u & \text{if } u < -A^0 / \gamma \end{cases}$$

If $A^0 > 0$, any positive $u$ will increase seller's effort from $A^0$ to $A^0 + \gamma u$. Therefore the buyer maximizes $A^0\left(a - bA^0\right) + \left(a - 2bA^0 - c^b / \gamma\right)\gamma u - b\gamma^2 u^2$ subject to $u \geq 0$. The KKT conditions are given by

$$\gamma\left(a - 2bA^0 - c^b / \gamma\right) - 2b\gamma^2 u \leq 0$$

$$\left[\gamma\left(a - 2bA^0 - c^b / \gamma\right) - 2b\gamma^2 u\right]u = 0$$

The second order condition is $-2b\gamma^2 < 0$. Solving for $u$ and replacing in the seller's best response function we obtain the optimal solution

$$u^* = \begin{cases} \dfrac{a - c^b / \gamma - 2bA^0}{2b\gamma} & if\ a - c^b / \gamma > 2bA^0 \\ 0 & if\ a - c^b / \gamma \le 2bA^0 \end{cases}$$

$$x^* = \begin{cases} \dfrac{a - c^b / \gamma}{2b} & if\ a - c^b / \gamma > 2bA^0 \\ A^0 & if\ a - c^b / \gamma \le 2bA^0 \end{cases}$$

If $A^0 \le 0$, the seller will exert a positive effort only if $u \ge -A^0 / \gamma$. Thus, the buyer finds $u$ that maximizes the net benefit $A^0\left(a - bA^0\right) + \left(a - 2bA^0 - c^b / \gamma\right)\gamma u - b\gamma^2 u^2$ subject to $u \ge -A^0 / \gamma$. Additionally, this net benefit must be positive. Otherwise, the buyer will provide no support. The KKT conditions are given by

$$\gamma\left(a - 2bA^0 - c^b / \gamma\right) - 2b\gamma^2 u + \lambda = 0$$

$$u + A^0 / \gamma \ge 0$$

$$\left(u + A^0 / \gamma\right)\lambda = 0$$

$$\lambda \ge 0$$

On the one hand, if $u = -A^0 / \gamma$, then $\lambda = -\gamma a + c^b$. This is feasible if $a \le c^b / \gamma$. However, $\pi^B\left(-A^0 / \gamma\right) = A^0 a < 0$, so the buyer will prefer to provide no support. On the other hand if $\lambda = 0$, then $u = \left(a - c^b / \gamma - 2bA^0\right) / 2b\gamma$, which is feasible if $a > c^b / \gamma$. However, $\pi^B\left(\left(a - c^b / \gamma - 2bA^0\right) / 2b\gamma\right) \ge 0$ only if $a - c^b / \gamma > \left(-4bc^b A^0 / \gamma\right)^{1/2}$. Thus

$$u^* = \begin{cases} \dfrac{a - c^b / \gamma - 2bA^0}{2b\gamma} & if\ a - c^b / \gamma > \left(-4bc^b A^0 / \gamma\right)^{1/2} \\ 0 & if\ a - c^b / \gamma \le \left(-4bc^b A^0 / \gamma\right)^{1/2} \end{cases}$$

$$x^* = \begin{cases} \dfrac{a - c^b / \gamma}{2b} & \text{if } a - c^b / \gamma > \left(-4bc^b A^0 / \gamma\right)^{1/2} \\ 0 & \text{if } a - c^b / \gamma \leq \left(-4bc^b A^0 / \gamma\right)^{1/2} \end{cases}$$

The optimal benefits are obtained by replacing the optimal effort and support in the net benefit functions (3.1) and (3.3).

*Proof of Proposition 3.2*

The symmetric equilibrium is given by $u_1 = u_2 = K / 2$. We can obtain $u_1^*$ and $u_2^*$ using the expression for $K$ obtained in equation (3.10). Thus, we obtain $x^*$ by substituting $u_1^*$ and $u_2^*$ in equation (3.8). The optimal net benefits can be obtained by substituting $x^*$, $u_1^*$ and $u_2^*$ in equations (3.7) and (3.9).

*Proof of Proposition 3.3*

From (3.17), we have a system of two equations. Solving for $u_1$ and $u_2$ we obtain $u_1^*$ and $u_2^*$. Then, we substitute these in equation (3.15) to obtain $x_1^*$ and $x_2^*$. Finally, we substitute the optimal efforts and supports in equations (3.13) and (3.16) to obtain the respective optimal net benefits for sellers and buyers.

*Proof of Proposition 3.4*

1. By directly comparing $x^*$ from equation (3.11) with $x_1^* + x_2^*$ from equation (3.18), we can determine that the effort in the shared-seller network is greater than the total effort in the separate-sellers network if $a \leq -2c^b / \gamma$. However, this is not possible given our assumption that $\tilde{K} > 0$.

2. From equations (3.11) and (3.18), we can determine that the effort in the shared-seller network is greater than the effort of each seller in the separate-sellers network if $a \geq \frac{4c^b}{\gamma}$. Additionally, the seller's net benefit function in both networks can be written as $\tilde{r}\left[1-(1+\alpha x)e^{-\alpha x}\right]$ (because $\delta = 0$ and $\hat{r} = \tilde{r}$). Because this function is increasing in the effort $x$, then the seller's net benefit is greater in the shared seller network relative to the separate-sellers network if $a \geq \frac{4c^b}{\gamma}$.

3. By direct comparison of the optimal buyer's net benefit from equations (3.12) and (3.19) we can determine that this net benefit is higher in the shared-seller network if $a \geq \frac{4c^b}{\gamma} + 36b\ln\left(\frac{\alpha\tilde{r}}{c^0}\right)$.

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# Biography

Alvaro Mendoza was born on October 5, 1981, in San Juan del Cesar, Guajira , Colombia. After receiving a full scholarship from the Colombian National Oil Company, ECOPETROL, he enrolled to The Department of Industrial Engineering of the Universidad de Los Andes, in Bogotá, Colombia, where he received his undergraduate and master degrees in Industrial Engineering in 2002 and 2004, respectively. After graduation, he started to work as an Instructor at the same university, combining teaching, research, and consulting tasks in the areas of Decision Analysis and Risk Analysis. Given his interest in these areas, he decided to apply to the PhD program in Business Administration at the Decision Sciences group of The Fuqua School of Business at Duke University. He is scheduled to complete his PhD in September of 2013 and will join the Deloitte Business Valuation - Decision Analysis group as a senior associate, pending successful defense of this dissertation.